

AN OPTIMIZATION PROCEDURE FOR ESTIMATING THE STOCK OF CAPITAL: APPLICATION TO TEN PRODUCTION SECTORS OF DUBAI

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Abstract

This paper develops and optimization procedure for estimating the capital stock using time series data on GDP, labor, and gross fixed capital formation. By minimizing the gap between observed and attainable GDP, subject to the constraint that capital grows according to the perpetual inventory formula, and GDP is related to employment and the capital stock through the Cobb-Douglas production function with constant returns to scale; the optimization procedure yields estimates of the capital stock, the production function parameters, the rate of capital retirement and depreciation, and the rate of total productivity growth.

Keywords: Stock of Capital, Production Sectors MENA Countries, Dubai, EAU

JEL Codes.

1. Introduction

The stock of capital can be measured either directly through surveys, published company accounts, balance of fixed assets, and administrative records (OECD, 2001); or indirectly through the Perpetual Inventory Method (PIM) introduced by Goldsmith (1952). In either case, the basic idea is to start with a benchmark year, cumulate past investments, and deduct assets that are no longer in use. However, because of the expense, practical difficulties, data and measurement problems associated with using direct methods to capital stock estimation, PIM is the most commonly used.

Its simplicity notwithstanding, PIM capital estimates are prone to error and are not insensitive to the different methods for estimating benchmark capital, and depreciation and retirement rates for assets. The literature is replete with attempts to refine those methods, especially with regards to capital replacement and depreciation (e.g., Epstein and Denny, 1980; Edward, 1983; Hernandez and Ignaci, 2005; Terregrossa, 1995). Still, without data on the amount of capital actually retired, the capital estimates are simply historical series of investment rather than capital stock (Miller, 1990). This makes their value to growth accounting questionable. By not knowing how much was invested to

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replace old capital and how much to augment the capital, production function estimates using PIM based capital estimates are not reliable (p. 67).

In this paper we propose an alternative indirect method that allows estimation of the stock of capital without extraneous information on benchmark capital, and retirement and depreciation, as required by PIM. The basic requirements to apply the method are time series data on GDP, employment, and gross fixed capital formation. Assuming real GDP is related to employment and the stock of capital through the usual Cobb-Douglas production function with constant returns to scale, and the stock of capital diminishes as a result of year-to-year wear and tear and increases by gross fixed capital formation, estimates of benchmark capital and the depreciation are obtained by minimizing the gap between observed GDP and potential GDP, with PIM imbedded into the optimization procedure. The result is a time series of the capital stock that is consistent with minimized GDP gap over the sample period along with estimates of the elasticity of production of labor, benchmark capital, the depreciation rate, and total factor productivity growth (TFP).

The details of the optimization procedure are outlined in the next section. The third section applies the optimization procedure to data from Dubai, United Arab Emirates. Summary and conclusions are in the final section.

2. Model

We start with the standard assumption that the stock of capital accumulates according to the PIM equation

$$K_t = (1 - \delta)K_{t-1} + G, \quad (1)$$

where t is time, δ is the (constant) depreciation rate, and G is real gross fixed capital formation. When benchmark capital K_0 and δ are known, repeated substitutions of lagged capital stock in equation 1 yields the capital stock time series

$$K_t = (1 - \delta)^t K_0 + \sum_{i=0}^{t-1} (1 - \delta)^i G_{t-i}, \quad (2)$$

where t is the end of the time period.

Next we follow convention and assume potential GDP is determined by labor, capital, and TFP through the Cobb-Douglas production function¹

$$GDP_t^{cap} = f(t)L_t^\alpha K_t^{1-\alpha}, \quad (3)$$

¹Among the major economic institutions that estimate potential GDP likewise are the International Monetary Fund (Adams and Coe, 1990), the OECD (Torres and Martin, 1990) and US Congressional Budget Office (CBO, 2001).

where, for each period t ,

$$\begin{aligned} GDP^{cap} &= \text{real GDP capacity,} \\ L &= \text{labor use,} \\ K &= \text{stock of capital,} \\ f &= \text{TFP,} \end{aligned}$$

and α is the elasticity of GDP with respect to labor use. Evolution of TFP is assumed to take the form

$$f(t) = \phi e^{\gamma \tau} \quad (4)$$

where ϕ is a scaling factor, γ is the annual percent change in TFP, and τ an index of technology.

However, the potential GDP that is pertinent in our case is not maximum *sustainable* output, but maximum *attainable* or *frontier* output (Oulton, 2004). In contrast to *sustainable* output, defined as “the level of real GDP in a given year that is consistent with a stable rate of inflation (CBO, 2001)”, *attainable* output is the level of real GDP in a given year that is consistent with most efficient utilization of labor and capital relative to full capacity output (Bansak *et al.*, 2007).

If we denote the gap between observed and frontier GDP by

$$\varepsilon_t = GDP_t^{cap} - GDP_t \geq 0. \quad (5)$$

we can construct the following optimization program to determine the elasticity of production with respect to labor (α), benchmark capital (K_0), the depreciation rate (δ), TFP growth (γ), and the scaling factor (ϕ)

$$\min_{\alpha, \delta, \phi, \gamma, K_0} \sum_{t=1}^T (\varepsilon_t)^2 = \sum_{t=1}^T (GDP_t^{cap} - GDP_t)^2$$

where

$$GDP_t^{cap} = \phi e^{\gamma \tau} L_t^\alpha K_t^{1-\alpha} \text{ for } \tau = t$$

and

$$K_t = (1-\delta)^t K_0 + \sum_{i=0}^t (1-\delta)^i G_{t-i},$$

subject to

$$GDP_t \leq GDP^{cap} \forall t$$

3. Application

In this section we illustrate the application of the optimization procedure using data on the 10 sectors that make up Dubai's Gross Domestic Product (GDP). The sectors are agriculture; mining and quarrying; manufacturing; electricity, gas, and water; construction; trade, restaurants, and hotels, transport, storage, and communication; Finance and insurance; real estate and business services; and social and personal services. The time series data on GDP, labor, and gross fixed capital formation (GFCF) for each sector for the 1985-2005 period are in Table 1, in the Annex. The sectors' respective parameter and capital stock estimates obtained through the optimization procedure are in Table 2, in the Annex. Tables 3 and 4 present a summary of Table 1 regarding the evolution of GDP and labor.

Table 3. Evolution of GDP by sector in Dubai (million UAE Dirham at 1995 prices)

Sector	1985	1995	2005
Agriculture	132	264	818
Mining and quarrying	44	67	87
Manufacturing	3418	4677	15820
Electricity, gas and water	719	813	1652
Construction	2065	3464	13054
Trade, Rest., Hotels	4253	7897	29230
Transport, Communications	1678	4490	13540
Finance and Insurance	1862	3409	12789
Real State, Business services	1496	4389	13248
Social and personal services	1064	1189	3351

Table 2. Evolution of Labor by sector in Dubai (number of people)

Sector	1985	1995	2005
Agriculture	3591	6985	11631
Mining and quarrying	300	530	815
Manufacturing	21813	47550	132093
Electricity, gas and water	4375	4300	8103
Construction	22815	69230	227251
Trade, Rest., Hotels	42388	96220	276211
Transport, Communications	21710	34867	74724
Finance and Insurance	4586	8999	16899
Real State, Business services	690	10975	29330
Social and personal services	34287	53131	105428

Taking agriculture as an example, the parameter estimates in table 2 are read as follows. The benchmark capital for this sector is 33 million UAE Dirham (AED) (1 US\$= 3.67 AED). In addition, assuming perfect competition and profit maximizing behavior in the sector, the share of labor is 27% and the share of capital is 73%. Total factor productivity growth in the sector grew at an annual rate of 0.52 percent from initial base of 0.73. The average annual rate of capital depreciation is 6%. The other sectors' respective parameter estimates can be read in a similar manner.

Agriculture is one of three sectors for which we were able to obtain reasonable estimates of the parameters without imposing additional constraints. The other two sectors are finance and insurance; and real estate and business services. For the rest of the sectors, additional constraints on the range of the parameters were needed to ensure the initial stock and, hence, the capital stock series are within an economically meaningful range. For example, the rate of capital retirement and depreciation in the mining and quarrying sector was fixed at an annual rate of 19 percent. In the case of manufacturing, the share of labor and the depreciation rate were fixed at 0.22 and 15%, respectively. An underlined value for a parameter in Table 2 means that parameter was fixed at that value.

As there are no other existing data on Dubai capital stock, it is not possible to tell how far off or close our estimates are to estimates that could have been obtained using other methods. But as we stated at the outset, the other methods are also prone to error and, above all, too costly to implement. In the light of that, the contribution of this paper is to add to the menu capitol stock estimation methods another method that researchers elsewhere can implement and, if possible, compare with other methods.

4. Conclusions

In this paper we have shown how optimization through linear programming can be used to estimate the capital stock given data on GDP, labor, and gross fixed capital formation. By minimizing the gap between observed and attainable GDP, subject to the constraint that capital grows according the standard perpetual inventory formula, and GDP is related to employment and the capital stock through the Cobb-Douglas production function with constant returns to scale; the optimization procedure yields the capital stock, the production function parameters, the rate of capital retirement and depreciation, and the rate of total productivity growth. The procedure is applied to data from Dubai, providing the first-ever estimates of the capital stock for the Emirate.

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TABLE 1 - GROSS DOMESTIC PRODUCT, LABOR, AND GROSS FIXED CAPITAL FORMATION BY SECTOR

YEAR	AGRICULTURE			MINING AND QUARRYING			MANUFACTURING			ELECTRICITY GAS & WATER			CONSTRUCTION		
	GDP	Labor	GFCF	GDP	Labor	GFCF	GDP	Labor	GFCF	GDP	Labor	GFCF	GDP	Labor	GFCF
1985	132	3,591	27	44	300	27	3,418	21,813	1,991	719	4,375	108	2,085	22,815	135
1986	139	3,860	21	45	312	28	3,390	22,042	1,807	745	4,428	159	2,156	23,547	149
1987	147	3,730	17	45	324	25	3,362	22,273	1,842	773	4,478	233	2,252	24,302	165
1988	155	3,802	13	48	337	24	3,334	22,608	1,492	801	4,530	341	2,351	25,081	182
1989	164	3,875	10	48	351	23	3,307	22,742	1,357	831	4,583	501	2,455	25,885	202
1990	173	3,949	8	49	365	22	3,280	22,860	1,235	862	4,636	736	2,563	26,715	223
1991	178	4,889	13	52	408	28	3,236	23,215	1,160	886	4,400	798	2,733	28,280	274
1992	218	4,908	18	51	444	30	3,684	26,050	1,083	978	4,540	880	2,778	29,250	335
1993	258	6,105	28	60	476	33	4,207	27,300	983	952	4,786	902	3,116	31,530	398
1994	259	6,774	40	68	500	35	4,431	44,670	858	956	4,242	908	3,301	65,360	454
1995	264	6,985	40	67	530	37	4,677	47,650	1,200	813	4,300	1,060	3,464	69,230	487
1996	293	7,127	30	69	543	38	5,180	50,110	1,530	849	4,315	1,056	3,645	70,900	521
1997	390	7,302	25	71	559	24	6,784	67,500	1,559	867	4,422	1,048	3,893	73,735	560
1998	435	7,301	41	74	581	28	7,292	74,200	1,623	885	4,669	1,004	4,214	73,850	600
1999	450	7,490	22	75	592	19	8,565	80,420	2,000	943	6,220	857	4,615	79,050	667
2000	484	7,640	26	78	680	23	9,660	87,550	2,994	1,002	6,300	1,173	4,862	83,850	695
2001	503	3,660	62	72	312	24	10,008	22,042	2,871	1,049	4,426	1,050	4,832	23,547	728
2002	522	3,730	58	74	324	24	10,551	104,858	3,276	1,099	6,482	938	5,074	173,150	655
2003	627	10,538	60	80	337	22	11,536	115,150	3,064	1,154	7,088	1,033	5,380	201,111	1,315
2004	767	10,987	109	85	870	27	14,884	123,680	4,422	1,509	7,160	1,636	11,691	210,800	1,760
2005	818	11,631	104	87	815	28	15,820	132,093	4,627	1,652	8,103	1,676	13,054	227,251	1,834

YEAR	TRADE, RESTAURANTS & HOTELS			TRANSPORT STORAGE & COMMUNICATION			FINANCE & INSURANCE			REAL ESTATE & BUSINESS SERVICES			SOCIAL @ PERSONAL SERVICES		
	GDP	Labor	GFCF	GDP	Labor	GFCF	GDP	Labor	GFCF	GDP	Labor	GFCF	GDP	Labor	GFCF
1985	4,253	42,388	290	1,678	21,710	876	1,862	4,586	77	1,496	690	157	1,064	34,287	26
1986	4,427	44,665	341	1,774	21,905	929	1,900	4,630	59	1,648	1,068	170	1,126	32,871	29
1987	4,809	47,065	402	1,878	22,101	986	1,939	5,086	45	1,816	1,645	184	1,196	31,513	32
1988	4,798	49,593	475	1,983	22,300	1,046	1,979	5,356	34	2,001	2,541	200	1,269	30,212	36
1989	4,994	52,258	580	2,087	22,500	1,111	2,019	5,640	26	2,205	3,924	216	1,346	28,984	40
1990	5,199	55,065	681	2,217	22,702	1,181	2,080	5,840	20	2,430	6,059	235	1,427	27,768	45
1991	5,903	45,800	816	2,364	24,398	1,350	2,339	6,195	25	2,611	6,619	437	1,480	48,030	51
1992	5,938	47,485	1,000	2,860	26,303	1,534	2,371	6,488	32	2,791	7,232	807	1,523	53,321	57
1993	7,093	50,087	1,191	3,344	28,395	1,694	2,626	6,401	39	3,294	7,900	1,449	960	60,375	63
1994	7,483	87,770	1,365	3,898	29,819	1,799	3,160	8,449	46	3,993	8,631	2,503	1,099	50,000	67
1995	7,897	96,220	1,687	4,480	34,867	2,410	3,409	8,899	72	4,389	10,975	2,464	1,189	53,131	80
1996	9,414	103,989	1,872	4,790	36,880	3,517	4,046	10,120	80	4,883	11,260	2,207	1,261	54,280	94
1997	10,705	114,298	2,133	5,155	36,850	3,393	4,450	11,366	99	4,919	11,340	2,423	1,520	58,890	105
1998	11,089	120,676	2,311	5,598	39,937	3,587	5,240	12,345	107	5,071	11,992	3,244	1,645	65,540	118
1999	11,489	152,410	2,467	6,231	40,336	4,317	5,534	12,351	131	5,525	12,520	4,159	1,815	75,750	192
2000	11,945	159,880	2,701	7,563	41,275	3,707	5,915	12,442	132	5,915	15,465	3,848	1,858	82,148	211
2001	12,472	44,665	3,077	8,543	21,905	4,342	7,249	4,830	324	6,048	1,066	3,873	2,146	32,871	230
2002	13,487	223,673	3,289	10,910	22,101	3,514	7,789	5,086	285	6,204	22,726	3,431	2,288	91,972	459
2003	14,282	248,525	3,846	12,414	68,057	5,457	8,227	14,525	289	6,475	24,267	3,140	2,377	97,050	664
2004	25,423	269,466	5,715	12,148	69,889	6,844	10,430	14,865	242	10,668	28,380	4,281	3,195	99,820	676
2005	29,230	276,211	6,276	13,540	74,724	7,004	12,789	16,899	267	13,248	29,330	4,419	3,351	105,428	1,042

Note: GDP and GFCF are in millions of 1995 AED

TABLE 2 - PARAMETER ESTIMATES AND CAPITAL STOCK ESTIMATES BY SECTOR

YEAR	AGRI-CULTURE			ELECTRI-CITY, GAS & WATER	CONST-RUCTION	TRADE, RESTAU-RANTS & HOTELS	TRANS-PORT, STORAGE, & COMMU-NICATION	FINANCE & INSURANCE	REAL ESTATE & BUSINES SERVICES	SOCIAL & PERSONAL SERVICES
	MINING & QUARRYING	MANU-FACTURING								
1985	58	178	7,312	2,008	1,040	2,922	4,135	111	17,507	87
1986	76	170	8,022	2,006	1,034	2,825	4,444	147	17,710	103
1987	88	163	8,461	2,078	1,043	2,804	4,763	161	17,927	120
1988	96	156	8,684	2,253	1,069	2,858	5,095	162	18,161	137
1989	100	150	8,739	2,574	1,110	2,989	5,442	155	18,411	157
1990	103	143	8,663	3,104	1,167	3,202	5,806	142	18,680	178
1991	109	142	8,524	3,654	1,266	3,537	6,286	138	19,152	202
1992	122	144	8,328	4,222	1,411	4,006	6,877	142	19,995	229
1993	143	150	8,062	4,786	1,597	4,597	7,540	152	21,481	258
1994	175	157	7,710	5,311	1,812	5,272	8,208	166	24,024	286
1995	204	164	7,754	5,936	2,027	6,149	9,387	204	26,533	323
1996	221	170	8,121	6,518	2,244	7,098	11,495	242	28,790	368
1997	233	162	8,462	7,045	2,467	8,166	13,165	291	31,267	418
1998	261	157	8,816	7,485	2,697	9,253	14,777	338	34,569	473
1999	267	146	9,494	7,743	2,959	10,331	16,878	399	38,793	595
2000	277	142	11,064	8,297	3,210	11,483	18,053	449	42,715	717
2001	323	139	12,275	8,683	3,457	12,837	19,687	680	46,668	839
2002	361	136	13,710	8,927	3,594	14,201	20,247	823	50,186	1,173
2003	400	132	14,717	9,246	4,369	15,917	22,667	941	53,421	1,661
2004	484	135	16,931	10,142	5,474	19,244	26,111	988	57,802	2,287
2005	560	137	19,019	11,007	6,487	22,633	29,198	1,050	62,330	2,987

PARAMETER ESTIMATES

K_0	33.00	186	6,260	2,065	1,065	3,097	3,834	43.00	17,317	73
α	0.27	0.28	<u>0.22</u>	0.52	0.69	0.39	<u>0.41</u>	0.20	0.10	0.39
β	0.73	0.72	0.78	0.48	0.31	0.61	0.59	0.80	0.90	0.61
φ	0.73	0.20	0.55	0.28	1.01	1.01	0.31	11.39	0.18	2.43
γ	0.52	4.48	1.62	-2.90	-5.54	<u>-3.02</u>	2.83	-2.64	0.38	-7.79
δ	6.00	<u>19.00</u>	<u>15.00</u>	<u>8.00</u>	<u>15.00</u>	<u>15.00</u>	<u>15.00</u>	20.76	-0.19	<u>15.00</u>

Note: Capital stock estimates are in millions of AED.