

LOCATING THE MOST IMPORTANT TRANSACTIONS OF THE ECONOMY BY AN ALTERNATIVE SENSITIVITY ANALYSIS: APPLICATION TO THE EMPLOYMENT STRUCTURE OF NINE EUROPEAN COUNTRIES

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Abstract.

The input-output structural analysis is a valuable tool when designing economic policies since it allows us to know which are the most important linkages and sectors of an economic system. The coefficients' sensitivity analysis is one of the major approaches of the structural analysis. With it we may measure how the output is influenced by changes in the coefficients. However, the classical sensitivity analysis, based on the Sherman & Morrison formula, shows several limitations. In this paper, an alternative approach based on linear programming and that deals with such challenges is proposed. The technique is then applied to an employment analysis in nine European countries.

JEL classification: C67, C61, J21.

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1. Introduction

Since the time Wasily Leontief formalized the input-output analysis, this framework has been broadly used to design and assess sectoral policies. The input-output framework provides the advantage of supplying a global as well as a detailed outlook of the technical and economic relationships of an economic system.

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The first methodological tool used by policy-makers was most likely the demand model, which relates final demand flows to their effects in the sectoral output. In addition to this, the possibility of building sectoral price vectors allowed for the development of input-output price models that relate the cost of primary factors to the final price of sectoral output. Nevertheless, the models above run on flow vectors (quantities and prices, respectively) with the underlying assumption that structural coefficients are fixed.

Several methodologies were later developed that focused on structural coefficients. The coefficients describe the economy's technical structure, as well as the economic factors' behaviour. Thus, by carrying out a detailed analysis of these coefficients we may uncover the main links and activities from the point of view of economic growth. This is the way in which structural analysis allows policy-makers map the economic transactions that contribute to achieve economic policy targets.

Two main approaches may be distinguished for the structural analysis within the input-output framework.

On the one hand, the structural decomposition analysis suggests a comparison between the information supplied by the input-output tables (IOT) of two or more periods. The changes in the output are decomposed into several factors as follows: the effect of the demand flows changes, the effect of changes in prices, and the effect of variations in structural coefficients. In this way, the changes in the structural relationships involving greater effects over the output can be found. Skolka (1988), Rose & Casler (1996), Dietzenbacher & Los (1998), and de Mesnard (2004) provide a more comprehensive background on this subject.

On the other hand, we could apply a change to the structural coefficients of a single IOT with the purpose of observing its effect on the output. On a broad sense, that is the so-called 'sensitivity analysis', which is the aim of this paper.

The sensitivity analysis has a traditional beginning through the well-know formula of Sherman & Morrison (1950), which calculates the effects over an inverse matrix caused by a change in an element of the initial matrix. The sensitivity analysis based on Sherman & Morrison requires a lesser volume of information than the structural decomposition analysis (only one IOT instead of at least two IOTs at

fixed prices, or at current prices but with the vector of prices available). Certainly, this is an advantage in a situation of information shortage¹. However, it has several limitations. In fact, our proposal tries to overcome some of them by means of linear programming tools, which provide a more flexible framework for the analysis.

The paper proceeds as follows. The next section presents an overview of the sensitivity analysis of coefficients and its limitations. In section 3 our alternative approach is developed. In section 4 our sensitivity analysis approach is applied to the employment structure of a set of European countries. Finally, section 5 points out the paper's conclusions.

2. The Sensitivity Analysis

As was explained above, sensitivity analysis techniques are based on the determination of how changes in structural coefficients affect the output of activity branches. This is how we are able to identify the most important coefficients of the economic system, as well as those whose minimal variations involve great changes in the output.

Traditionally, Sherman & Morrison's (1950) formula has been considered as one of the main sources of the foundations of these techniques. This formula calculates the effects over an inverse matrix, caused by a change in an element of the initial matrix:

$$\Delta b_{ij} = -\frac{b_{ik}b_{lj}\Delta a_{kl}}{1 + b_{lk}\Delta a_{kl}} \quad (1)$$

where: a_{kl} element of the original matrix $\mathbf{A}$

b_{ij} element of the inverse matrix $\mathbf{B} = (\mathbf{A})^{-1}$

Evans (1954) applies the formula to the input-output framework. From this contribution, a set of authors have developed many operative proposals with the purpose of identifying the most important coefficients of the economic system (that is, those involving a high sensitivity of the output). Jílek (1971), Jensen &

¹ Both approaches are complementary, anyway.

West (1980), Maas (1980), West (1982), Schintke & Stäglin (1988), Hewings et al. (1989), Sonis & Hewings (1992), Songlin & Gould (1991), Sonis et al. (1996) and Siebe (1996), among others, could be quoted. All of them assess the changes on one or more coefficients.

In order to explain the idea underlying the sensitivity analysis, we are going to define the basic relations of an IOT. First, the sum of the sales of the domestic production of an activity branch i represents its output:

$$\sum_j x_{ij} + \sum_k y_{ik} = w_i \quad (2)$$

where:

- x_{ij} is the element of the $\mathbf{X}$ (nxn) intermediate transaction matrix that shows the purchases by the j^{th} branch of the goods produced by the i^{th} branch,
- y_{ik} is the element of the $\mathbf{Y}$ (nxk) final demand matrix that shows the production of the i^{th} activity branch, which is purchased by the k^{th} final demand aggregate, and
- w_i is the output of the i^{th} activity branch that belongs to the $\mathbf{W}$ (nx1) vector of output.

The concept of output leads to the definition of technical coefficient, which is the ratio between each element of the intermediate transaction matrix and the output of the corresponding activity branch (column):

$$a_{ij} = \frac{x_{ij}}{w_j} \quad \text{with } i=(1,2,\dots,n) \text{ and } j=(1,2,\dots,n) \quad (3)$$

The matrix of technical coefficients $\mathbf{A}$ is thus a (nxn) matrix.

Based on the identity, (2) the following matrix expression can be established:

$$\mathbf{W} = \mathbf{X}\mathbf{U}^X + \mathbf{Y}\mathbf{U}^Y \quad (4)$$

where:

- $\mathbf{U}^X$ is the unitary vector (nx1), and
- $\mathbf{U}^Y$ is the unitary vector (kx1)

If we include the $\mathbf{A}$ matrix in (4):

$$\mathbf{W} = \mathbf{A}\mathbf{W} + \mathbf{Y}\mathbf{U}^Y \quad (5)$$

This expression can be rewritten obtaining the well-known demand model:

$$W = (I - A)^{-1} Y U^Y \quad (6)$$

where: $(I-A)^{-1}$ is Leontief's inverse matrix.

A technical coefficient will be important if a change in it causes serious changes in the output of the activity branches. Obviously, expression (1) is applied to (6) for obtaining these changes in the output since the variation in the coefficient extends its effect throughout the changes in the Leontief's inverse matrix that it involves. This sort of techniques based on Sherman & Morrison's are broadly applied in the structural change assessment. However, they have several limitations that should be pointed out. Firstly, the output is taken into account as reference variable for the coefficients' sensitivity analysis. Instead, the policy-maker could be interested in assessing the changes in the economic structure in terms of another variable with economic, social or environmental interest (employment, or CO2 emissions, for instance). In this sense, a generalization of the analysis would be desirable. Secondly, we are assuming that structural coefficients and technical coefficients are the same.

Nevertheless, not only is economic structure defined by the technical relations between activity branches, but also by the final demand relations between these activities and the demand aggregates: private consumption, public consumption, capital stock formation or exports.

Therefore, the analysis should be extended to the final demand relations matrix. Both of these limitations are taken into account in our approach². Such an approach is based on the multiple and iterative solution of a linear programming model. A set of indexes then allows us to classify the structural coefficients, including the final demand ones, in terms of the chosen reference variable. The foundations of the proposal are described in the following section.

² For the study of other limitations, see Schnabl (2003).

3. The Sensitivity Analysis Based on Linear Programming

Our approach for measuring the sensitivity of structural coefficients is based on the specification and iterative solution of a mathematical programming model. This model provides a more flexible framework in order to classify the transactions of an economic system according to a general variable taken as reference. Such classification also includes the transactions of the final demand.

Dantzing (1949), Dorfman, Samuelson & Solow (1958) and Matuszewski et al. (1964), among others, made pioneering efforts in the use of mathematical programming within the input-output framework. More recently, the ANAIS adjustment and projection model (Tarancón & Río 2005) can be considered as the starting point of our methodology. In the following subsections, the model specification and its iterative solution process are shown.

3.1. Model for the Identification of the Most Important Transactions (MITs). We are going to start by setting the reference variable (RV). We will assess the changes in the structural coefficients through it. Then, if we define the e ($n \times 1$) vector as the RV share between the n activity branches (sectors) of the economic system, we also be able to define the C diagonal matrix as the sectoral RV amount per unit of output. The elements of the main diagonal will be:

$$c_{jj} = \frac{e_j}{w_j} \quad (7)$$

If we integrate (7) into (5), we can then calculate the RV amount based on the final demand vector:

$$E = C(AW + YU^Y) \quad (8)$$

On the other hand, aggregate k can be defined as the sum by columns of the components of the final demand matrix:

$$g_k = \sum_i y_{ik} \quad (9)$$

Thus, the final demand coefficients can be calculated as follows:

$$h_{ik} = \frac{y_{ik}}{g_k} \quad \text{with } i=(1,2,...,n) \text{ y } k=(1,2,...,m) \quad (10)$$

This way, the matrix of final demand coefficients $\mathbf{H}$ will be a (nxm) matrix. If this matrix is included in (8), then:

$$\mathbf{E} = \mathbf{C}(\mathbf{A}\mathbf{W} + \mathbf{H}\mathbf{G}) \quad (11)$$

where : $\mathbf{G}$ is components vector (mx1) of the final demand.

The previous expression can also be detailed for each activity branch:

$$e_i = \sum_{j=1}^n c_{ij} a_{ij} w_j + \sum_{k=1}^m c_{ik} h_{ik} g_k \quad \forall i \quad (12)$$

The total quantity of RV units from the production system will be:

$$e_{tot} = e_1 + e_2 + \dots + e_n \quad (13)$$

If we assume there is an increment in the above quantity, the amount of RV for the period (1) will be:

$$e_{tot(1)} = e_{tot(0)} + \Delta e_{tot(0)} \quad (14)$$

If it is further assumed that this increment is due to the change of model's structural coefficients (both technical and final demand coefficients), then the following applies:

$$e_{i(1)} = c_{ii(0)} \left(\sum_{j=1}^n a_{ij}^* w_{j(0)} + \sum_{k=1}^m h_{ik}^* g_{k(0)} \right) \quad \forall i \quad (15)$$

The equalities of the system of equations in (15) will not be met, at least not for all n activity branches, since a change in the coefficients of matrix $\mathbf{C}$ would be needed. Nevertheless, a set of supplementary variables sv_* will be added which will pick up the errors in the system after the new RV level:

$$a_{ij}^* \leq a_{ij(0)} + sv_{a_{ij}^{\sup}} \quad (16)$$

$$h_{ik}^* \leq h_{ik(0)} + sv_{h_{ik}^{\sup}} \quad (17)$$

In all cases, the supplementary variables will meet the following condition:

$$sv_* \geq 0 \quad (18)$$

The minimisation of the sum of all supplementary variables can now be proposed as the model's objective:

$$\min \sum sv_* \quad (19)$$

Subject to the following conditions: (13), (14), (15), (16), (17), and (18).

The model will be solved through an iterative procedure. In each of the iterations, the objective's sensitivity to the changes in each structural coefficient will be measured. This procedure is described in the following subsection.

3.2. The Sensitivity Analysis Procedure. When the supplementary variables are introduced, then a feasible solution for the model is ensured, allowing us to analyse the sensitivity of the original coefficients. In order to do so, restrictions (16) and (17) will be modified in an iterative way, relaxing their variability limits (values of each coefficient at period 0, that is, $a_{ij(0)}$ or $h_{ik(0)}$). This way we may identify the modifications of the original coefficients that contribute to the objective's minimization (19). The information provided by this iterative process proves essential in order to determine which coefficients are most sensitive to the change in the global RV level proposed in (14). Let's consider an iteration whereby the limit of a coefficient is relaxed in the corresponding restriction (16) or (17), so that it can change in order to contribute to the objective (19). After solving the model, if a small change in the limit causes a large reduction in the objective, then we will consider the coefficient very important, as it contributes to the recovery of the balance of equations system (15) after the new RV level. On the contrary, a coefficient can be considered non-important if a large modification of its value is proposed but only a small reduction in the objective (19) is obtained. In order to identify and quantify coefficients' importance level, we will define the following elasticity indexes:

$$\delta_{ij}^a = \frac{\frac{obj_{bas} - obj_{ij}}{obj_{bas}}}{\frac{a_{ij(0)} - a_{ij}^*}{a_{ij(0)}}} \quad (20)$$

where:

obj_{bas} is the value of objective (19). In this case, variability limits of the coefficients imposed in (16) are not relaxed.

obj_{ij} is the value of objective (19). However, in this case variability limit of the a_{ij} coefficient imposed in (16) are relaxed.

A high value in δ_{ij}^a means that the level of the RV is highly sensitive to the changes in the coefficient, and therefore it is considered important.

We can also build elasticity indexes corresponding to the coefficients of the final demand (h_{ik}). In this case, the variability limits that will be relaxed match up with (17):

$$\delta_{ik}^h = \frac{\frac{obj_{bas} - obj_{ik}}{obj_{bas}}}{\frac{h_{ik(0)} - h_{ik}^*}{h_{ik(0)}}} \quad (21)$$

The most important economic transactions (MIT) in terms of the RV are depicted by coefficients a_{ij} and h_{ik} with high elasticities.

In the following section, an empirical application of the technique is discussed. More specifically, we will apply the sensitivity analysis to the case of employment in a set of European countries, emphasizing the main differences and similarities among their structures.

4. An Application: The Employment Structure of Nine European Countries

As we have seen above, the sensitivity analysis based on Sherman & Morrison's formula provides the technical coefficients involving the most important transactions from the sectoral output point of view. However, policy-makers might be interested in stimulating most important transactions from within a social framework. That is precisely the case of employment³ (Prakash & Balakrishnan, 2005). Hence, the employment level will be set here as the reference variable (RV). Furthermore, the economic transactions that

³ For an example of environmental application, see Tarancón & del Río (2004).

contribute to the targets fixed in employment terms not only can take place through intersectoral trade, but also through sales to the final demand components. In fact, these transactions are often a strong incentive over employment targets. This stimulus can be direct as well as indirect, since final demand purchases yield the production of goods and services in the related activities that lead to changes in the overall employment amount⁴. Usually, scholars consider the final demand as an exogenous factor, focusing on the technical relations between sectors and proposing final demand vectors *a priori* to impact on the inter-sectoral structure.

On the contrary, we suggest including the final demand factor within the sensitivity analysis rather than considering it on an exogenous way. Hence, we will locate the most important final demand transactions from the employment viewpoint. Starting from these previous considerations, a sensitivity analysis of the all-domestic structural coefficients of nine European 1995-IOTs (Austria, Belgium, Denmark, Finland, Germany, Italy, the Netherlands, Spain, and Sweden) has been carried out. The IOTs have been extracted from Eurostat Database and aggregated to 30 activities for their compatibility with the Eurostat employment vectors (see Appendix). For each of the nine countries, an increment of 1% in the overall employment level is proposed, applying the procedure shown in section 3.

Then, the most important transactions (MIT) have been pointed out as those whose associated structural coefficients show high elasticity indexes (20) or (21). Table 1 shows the *top five* intermediate and final demand transactions (related to the elasticity index criteria) for each country. The ‘Elasticity’ column represents the corresponding sensitivity index δ_{ij}^a (or δ_{ik}^h).

⁴ We assume the stimulus should be sustained and intensive enough, anyway.

Table 1a. Sensitivity Analysis. Top-Ten Important Transactions in Terms of Their Contribution to Employment.

Austria.					
Technical coefficients			Final Demand Coefficients		
Transaction (seller -> buyer)		Elasti- city	Transaction (seller -> buyer)		Elasti- city
1 Agriculture	5 Manuf. food	8.5806	21 Trade	FCEH	7.3179
1 Agriculture	8 Manuf. wood	1.2317	26 Public Admin.	FCEG	6.7110
20 Construction	25 Business Act.	0.7893	20 Construction	GFCF	5.1706
25 Business Act.	21 Trade	0.6895	22 Hotels&Rest.	FCEH	4.5958
25 Business Act.	25 Business Act.	0.6809	28 Social work	FCEG	4.5465
Belgium.					
Technical coefficients			Final Demand Coefficients		
Transaction (seller -> buyer)		Elasti- city	Transaction (seller -> buyer)		Elasti- city
25 Business Act.	25 Business Act.	2.0614	26 Public Admin.	FCEG	9.8127
20 Construction	20 Construction	1.8944	27 Education	FCEG	8.0938
1 Agriculture	5 Manuf. food	1.8566	21 Trade	FCEH	6.0184
25 Business Act.	21 Trade	1.4650	28 Social work	FCEG	4.7978
23 Transport	23 Transport	1.0978	25 Business Act.	FCEH	3.4811
Denmark.					
Technical coefficients			Final Demand Coefficients		
Transaction (seller -> buyer)		Elasti- city	Transaction (seller -> buyer)		Elasti- city
1 Agriculture	5 Manuf. food	2.4813	28 Social work	FCEG	12.4939
23 Transport	23 Transport	0.9673	26 Public Admin.	FCEG	7.0456
21 Trade	20 Construction	0.9650	21 Trade	FCEH	6.7760
23 Transport	21 Trade	0.9233	27 Education	FCEG	6.1925
25 Business Act.	21 Trade	0.8252	20 Construction	GFCF	3.5595

Source: own elaboration

Table 1b. Sensitivity Analysis. Top-Ten Important Transactions in Terms of Their Contribution to Employment

Finland.					
Technical coefficients			Final Demand Coefficients		
Transaction (seller -> buyer)		Elasticity	Transaction (seller -> buyer)		Elasticity
1 Agriculture	5 Manuf. food	2.5357	28 Social work	FCEG	9.9494
1 Agriculture	8 Manuf. wood	1.4062	26 Public Adm.	FCEG	6.0885
1 Agriculture	1 Agriculture	1.3276	21 Trade	FCEH	6.0724
9 Pulp, paper	9 Pulp, paper	0.9004	27 Education	FCEG	5.1486
1 Agriculture	9 Pulp, paper	0.8711	20 Construction	GFCF	4.0104
Germany.					
Technical coefficients			Final Demand Coefficients		
Transaction (seller -> buyer)		Elasticity	Transaction (seller -> buyer)		Elasticity
1 Agriculture	5 Manuf. food	1.6319	21 Trade	FCEH	8.2516
25 Business Act.	25 Business Act.	1.2141	26 Public Adm.	FCEG	6.9531
23 Transport	23 Transport	1.1873	20 Construction	GFCF	6.7383
21 Trade	21 Trade	0.8960	28 Social work	FCEG	6.2948
20 Construction	25 Business Act.	0.7941	27 Education	FCEG	4.0161
Italy.					
Technical coefficients			Final Demand Coefficients		
Transaction (seller -> buyer)		Elasticity	Transaction (seller -> buyer)		Elasticity
1 Agriculture	5 Manuf. food	2.8725	21 Trade	FCEH	9.8019
6 Textile prods.	6 Textile prods.	1.1142	26 Public Admin.	FCEG	6.4179
28 Social work	28 Social work	0.9538	27 Education	FCEG	6.1410
25 Business Act.	21 Trade	0.9005	20 Construction	GFCF	4.7466
25 Business Act.	25 Business Act.	0.8456	30 Households	FCEH	4.3598

Source: own elaboration

Table 1c. Sensitivity Analysis. Top-Ten Important Transactions in Terms of Their Contribution to Employment

The Netherlands.					
Technical coefficients			Final Demand Coefficients		
Transaction (seller -> buyer)		Elasti- city	Transaction (seller -> buyer)		Elasti- city
1 Agriculture	5 Manuf. food	1.6861	28 Social work	FCEG	8.2263
25 Business Act.	25 Business Act.	1.4769	21 Trade	FCEH	7.2738
20 Construction	20 Construction	1.3875	26 Public Adm.	FCEG	5.4422
25 Business Act.	21 Trade	1.0346	27 Education	FCEG	5.0611
21 Trade	21 Trade	0.7597	25 Business Act.	FCEH	4.5755
Spain.					
Technical coefficients			Final Demand Coefficients		
Transaction (seller -> buyer)		Elasti- city	Transaction (seller -> buyer)		Elasti- city
1 Agriculture	5 Manuf. food	4.0985	21 Trade	FCEH	9.2099
20 Construction	20 Construction	1.5435	26 Public Adm.	FCEG	8.7244
21 Trade	20 Construction	0.8161	20 Construction	GFCF	6.0610
13 Non-metal	20 Construction	0.7665	22 Hotels&Rest.	FCEH	5.4027
20 Construction	25 Business Act.	0.7338	30 Households	FCEH	4.6127
Sweden.					
Technical coefficients			Final Demand Coefficients		
Transaction (seller -> buyer)		Elasti- city	Transaction (seller -> buyer)		Elasti- city
20 Construction	25 Business Act.	1.1855	28 Social work	FCEG	13.4317
1 Agriculture	5 Manuf. food	1.1332	27 Education	FCEG	8.4242
25 Business Act.	25 Business Act.	1.1169	26 Public Adm.	FCEG	6.3968
23 Transport	23 Transport	1.0459	21 Trade	FCEH	5.3574
29 Other services	25 Business Act.	0.7847	25 Business Act.	FCEH	3.3326

Source: own elaboration

On the left side of table 1 we can see that the most important intermediate transactions in the majority of the countries, from an employment point of view, are the sales from the primary sector (activity 1) to branch 5 *Manufacture of food products beverages and tobacco*. For instance, the elasticity index (21) for this transaction is,

in the case of Austria, 8.58. This value means that a change of 1% in the corresponding coefficient would lead to, *ceteris paribus*, a reduction of 8.54% in the objective (20), assuming a 1% rise in the overall employment (14).

The right side of the table shows the most important transactions in the final demand matrix. The results can be interpreted in the same manner. For instance, in the case of Austria, an increase of 1% in the coefficient that represents the sales share from activity branch 21 *Wholesale and retail trade...to FCEH Final consumption expenditures by households* allows the system to reduce the objective (20) by 7.31%, assuming a 1% rise in the overall employment (14).

In order to systematize the comparison of the results of the nine countries, the MITs have been stressed by applying two filters to matrixes **A** and **H**. The filters, δ^g and δ^h respectively, were set on indexes (20) and (21). If the associated sensitivity index $\delta^g_{ij} > \delta^g$, then coefficient a_{ij} is considered important, and the economic transaction that it represents is therefore one of the most important (MIT) of the economic system. In the case of final demand coefficients, if the associated sensitivity index $\delta^h_{ik} > \delta^h$, the aforementioned conclusion regarding the final demand transaction ik applies.

The complexity of the analysis will be increased by low filters, since they will include more transactions, whereas high δ filter values will allow us to focus on those relationships leading to the highest changes in the overall employment level. The filter selection is obviously arbitrary. We have tried to fix a set of filters that provide a balanced outlook of the economic relationship, that are neither too high for the information supplied to be too poor, nor too low for the results to show identical relations for each country (since nearly all realations surpass the filter). Regarding the elasticity indexes, we may notice how the final demand indexes (22) are in general terms higher than the intermediate transactions indexes. This is due to the fact that final demand transactions lie in the beginning of the economic system transactions chain, and therefore involve intermediate transactions. Thus, the effect of the final demand transactions over the employment level will be usually higher. Given this, we have applied two different filters to avoid focusing

We have counted the number of countries where each transaction surpasses its filter. The transactions that are important in more than half of the countries match with a non-empty cell in the new matrix. Thus, this matrix shows the *typical structure of the most important transactions* (MITTS matrix) for the nine countries as a whole. As far as the Intermediate Demand (a_{ij} coefficients), we can notice how the most important transactions (MIT) are concentrated in the sales (rows) for sectors 21 *Wholesale and retail trade...*, 23 *Transport, storage and communication*, and 25 *Real estate, renting and business activities* to sectors 20 to 25. Furthermore, we can observe the growth in importance of self-purchases. Finally, we may emphasize that the important transactions from sector 21 *Wholesale and retail trade...* extend its sales to manufacturing activities. Focusing on the Final Demand, we may notice as main similarities among the countries the relevance of the sales from sectors 21 to 23, 25, 28, 29 to households' consumption, as well as the sales from sectors 26 to 28 to the Government's consumption. Moreover, we could underline the sales from activity 20 *Construction* to the gross fixed capital formation, the financial brokerage indirectly measured from sector 24 *Financial intermediation*, and the exports from sectors 21 *Wholesale and retail trade...*, as well as from 15 *Manufacture of machinery and equipment n.e.c.*

Table 2a. Differences from the MIT structures regarding to the MITTS

Country	Intermediate Demand	Final Demand
Austria	(+) sales from 1 Agriculture (-) sales from 21 Trade and 25 Business act.	(+) sales from 1 Agriculture and manufacturing sectors. (-) FCEH from 22 Hotels... (-) FCEH and EXP from 23 Transport
Belgium	(+) sales from services sectors to manufacturers.	(+) EXP from manuf. sectors. (+) Sales from Business act.
Denmark	(+) sales from 21 Trade to manufacturing sectors., and (-) to 17 Manuf. Transport equip., 22 Hotels and 25 Business act. (+) sales from 22 Hotels to the services sectors. (+) sales from 20 Construction to 25 Business act.	Very homogeneous structure regarding the typical structure (MITTS).

With the aim of providing the main differences between the structures of each country, we have compared results in each country with the MITTS matrix. Based on this simple operation, tables 2a and 2b have been built, summarizing the main results.

Table 2b. Differences from the MIT structures regarding to the MITTS

Country	Intermediate Demand	Final Demand
Finland	(+) sales from 1 Agricultur, manuf. sectors energy sector. (+) sales from 21 Trade to tertiary sector, and (-) to the manufacturers. (+) sales from 23 Transport (-) sales from 25 Business act. to services.	(+) EXP from 9 Pulp, paper and 16 Manuf. Electrical equipment (-) EXP from 21 Trade...
Germany	(+) sales from 25 Business act. to manufacturing sectors, and (-) to the services activities.	(+) EXP from 16 Manuf. Electrical equipment and 17 Manuf. Of transport equipment. (-) EXP from 23 Transport
Italy	(+) Manufacturing activities (most specially the self-purchases). (+) sales from 21 Trade... to manufactures of textil, leather, pulp and paper and 11 Chemicals.	(+) FCEH from 1 Agriculture 5 Manuf. food 6 Manuf. Textil and 27 Education. (-) FCEH from 28 Social work
The Netherlands	(+) sales from 25 Business act.	(+) Exporting activities.
Spain	(+) Transactions among manuf. activities. (+) sales from 23 Transport (+) purchases from 20 Construction. (-) Transactions among services sectors.	(-) Exporting activities. (+) Household sales.
Sweden	Very homogeneous structure regarding the typical structure (MITTS).	(+) FCEG from sectors involving social and public services. (+) Exporting activities.

Notes to tables 2a and 2b: (+) means a higher weight than in the majority of the countries (MITTS).; whereas (-) means a lower weight. Source: own elaboration

Considering the findings seen above, in the following section a set of limitations regarding the technique and conclusions are set forth.

5. Conclusions.

In this paper, an alternative approach for locating the most important transactions (MITs) of an economic system are presented. To reach this aim, we propose solving a linear programming model through an iterative procedure. The model connects the input-output table with a reference variable by means of the balanced input-output relations.

This methodology provides two advantages that try to overcome both limitations carried by the classical methodologies of sensitivity analysis. On the one hand, it allows us to consider a criteria that is different to the output (for instance employment, CO₂ emissions, and so on) for the classification of the most important transactions. On the other hand, it allows extending the analysis to the Final Demand. The approach is based on changing the overall level of the reference variable, and locating the structural coefficients whose minimal variations lead to a balanced situation assuming that new level.

Nevertheless, we must emphasize that this proposal is also prone to certain limitations that should be pointed out. First, the linear model implies linear relations, which should of course be assumed. For instance, in our empirical application, a fixed relation between an increment in the output and an increment in the number of employees is assumed. This is certainly not true. Therefore, the linear description of the reality is only true if we take into account the idea of infinitesimal variations (in fact, that is the assumption underlying the concept of ‘sensitivity analysis’).

Moreover, the exogenous quality of the value of the importance filters should be taken into account, since it obviously involves a certain degree of arbitrariness. Hence, a criterion for setting an endogenous filter (or a filter justified from a theoretical point of view) should be achieved.

Assuming all these limitations, the methodology has been applied to nine European countries, locating the most important transactions (MITs) from the point of view of their contribution to the employment level.

With regard to the similarities among the structures of MITs, we can point out that the majority of them are concentrated in the sales from the activities of the tertiary sector (services) to themselves, as well as to the Final Demand. This is probably due to the fact that they are labour intensive activities. We can also emphasize the importance of the self-purchases, and of the sales from sector 21 *Wholesale and retail trade...* to a broad group of sectors.

Focusing on the differences among countries, a certain tendency to extend the location of MITs to the sales of the manufacturing activities within the greatest economies –in terms of Added Value– (i. e. Germany, Italy, Spain) is detected. Finland, Sweden and the Netherlands show a higher number of MITs in the trade among the public and social services (sectors 26 to 29), whereas in Austria, Germany, and Spain the opposite occurs. Finally, we can notice a higher number of MITs in the exports of the smallest countries, that is, Austria, Belgium as well as Sweden, which might involve a high level of dependence from foreign trade.

Our aim is not to provide policy recommendations for improving European employment level, but rather proposing a useful tool for locating the economic structure's "hot-spots" in terms of a worth political variable (employment). Nevertheless, we will suggest several ideas related to the results that could be taken into account in the design of a joint sectoral employment policy.

According to that, it could be stressed that, if the MIT patterns are homogeneous for all countries, a joint policy of domestic demand stimulus or foreign trade will involve balanced effects for all the countries. In this case, the transactions of the MITTS matrix could be taken as reference for drawing up the strategy of global employment improvement.

Nevertheless, if the MIT structure is very heterogeneous between countries, a joint strategy could lead to a rise of employment disparities. In this situation, a set of national specific strategies should be adopted.

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Appendix. R-31 Activity Branches and Final Demand Aggregates.

1	a Agriculture, hunting and forestry
2	b Fishing
3	ca Mining and quarrying of energy producing materials
4	cb Mining and quarrying except energy producing materials
5	da Manufacture of food products; beverages and tobacco
6	db Manufacture of textiles and textile products
7	dc Manufacture of leather and leather products

8	dd Manufacture of wood and wood products
9	de Manufacture of pulp, paper and paper products; publishing and printing
10	df Manufacture of coke, refined petroleum products and nuclear fuel
11	dg Manufacture of chemicals, chemical products and man-made fibres
12	dh Manufacture of rubber and plastic products
13	di Manufacture of other non-metallic mineral products
14	dj Manufacture of basic metals and fabricated metal products
15	dk Manufacture of machinery and equipment n.e.c.
16	dl Manufacture of electrical and optical equipment
17	dm Manufacture of transport equipment
18	dn Manufacturing n.e.c.
19	e Electricity, gas and water supply
20	f Construction
21	g Wholesale and retail trade; repair of motor vehicles, and household goods
22	h Hotels and restaurants
23	i Transport, storage and communication
24	j Financial intermediation
25	k Real estate, renting and business activities
26	l Public administration and defence; compulsory social security
27	m Education
28	n Health and social work
29	o Other community, social, personal service activities
30	p Household activities
FISIM	Financial brokerage indirectly measured
FCEH	Final consumption expenditures by households
FCEN	Final consumption expenditures by non-profit organisations serving households
FCEG	Final consumption expenditures by government
GFCF	Gross fixed capital formation
CIV	Changes in inventories and valuables
EXP	Exports