

Is the economy a complex system in eternal disequilibrium?*

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Abstract

This article presents a general discussion of some of the reasons for believing that an economy would be better framed in the theory of complex (organic) systems than in the theory of mechanical systems of the dominant neoclassical school. Complex systems are characterized here by heterogeneous units that interact with each other, with non-linear trajectories, positive feedbacks, co-creation of regularities, non-ergodicity of the system and constant evolution. Financial and urban systems are analyzed as examples of economic problems that present these characteristics.

Keywords: real world economics; disequilibrium; collective decision making.

JEL: B590; D5; D79.

¿Es la economía un sistema complejo en eterno desequilibrio?

Resumen

Este artículo presenta una discusión general sobre algunas de las razones que se tienen para creer que una economía estaría mejor enmarcada en la teoría de los sistemas complejos (orgánicos) que en la teoría de los sistemas mecánicos de la escuela neoclásica dominante. Los sistemas complejos se caracterizan aquí por unidades heterogéneas que interactúan entre ellas, con trayectorias no-lineales, retroalimentaciones positivas, co-creación de regularidades, no-ergodicidad del sistema y constante evolución. De manera particular, se analizan los sistemas financieros y urbanos como ejemplos de problemas económicos que presentan estas características.

Palabras clave: economía del mundo real; desequilibrio; toma colectiva de decisiones.

Clasificación JEL: B590; D5; D79.

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Introduction

Although they have been present since the ancient Greeks, it is only in recent decades that complex systems have come to take a prominent role as an alternative paradigm in the natural sciences, also permeating the study of social and economic systems. The general idea of this important (and very particular) type of system is that it is made up of a large set of heterogeneous units at the micro level, governed by certain *non-linear* rules of behavior, which are allowed to interact and develop dynamically over time, to see how certain regularities (e.g., patterns or structures) *emerge* spontaneously (without central control) over time. The area of study of these specific systems is called Complex Systems Theory or, in one word, *Complexity*.

In the so-called “hard sciences”, Complexity constituted an epistemological disruption from the dominant paradigm since the 17th century of Galileo, Descartes, Newton, and Leibniz. At that time, attempts were made to understand macro aggregate systems by reducing them “linearly” to the study of their micro-units (from the simplest to the most complicated), which, in fact, worked to some extent. However, for the complex systemic view, this is not the way to go when seeking to understand numerous phenomena, because the intricate non-linear dynamic behavior of such a system is qualitatively and statistically different from the behavior of its component units: there, the interactions of the micro-units do not occur uniformly, but through different “hierarchies” or “levels”, each governed by intrinsically distinct emergent dynamics.

For example, Nature recognizes these hierarchies by separating molecules from cells; the latter from organisms; the latter from species; and, finally, the latter from societies, having in each hierarchy or level, dynamics that operate in a totally different way. In the biological environment, it is precisely this behavior with hierarchical levels that promotes a high level of *fitness* (survival and reproductive possibility) of the species, by taking the best advantage of the environment (Deichmann, 2017; Krugman, 1996).

It is in this line that Complexity seeks to understand the economy as a dynamic system in eternal disequilibrium and integrated as well with sociology and the natural environment (Prigogine & Stengers, 1997). After all, it is in these contexts that we (human beings) solve the problem of survival, reproductive possibility, and *well-being*.

Therefore, we must be aware that not only humans have an economy: all species (ants, bees, fishes, birds, plants, etc.) do! Understanding this is fundamental to the non-anthropocentric approach that guides the study of complex economic systems (Bassett & Claveau, 2018). Therefore, the following definition of Economics is at the very heart of this vision: Economics is the study of the different mechanisms by which different species (humans, ants, bees, fishes, birds, plants, etc.) solve their problems of survival and *well-being*, in correlation with other species and the natural environment.

It should be noted here that, in human beings, *well-being* is understood to have a multidimensional definition suggested by a commission led by Sen et al. (2010), which has identified the simultaneous consideration of the standards of material means of life (income, consumption, wealth), as well as health, education, personal activities, political voice, social connections and relationships, the environment, and economic and physical security.

On the systemic view in the history of economic thought

The systemic notion of complexity has been around (in one form or another) since Political Economy as a science was founded by the first classical economists of the 18th century. For example, the fundamental problem of economic liberalism posed by Smith (1776/1969) as “the invisible hand” –that is, understanding the emerging fundamental mechanisms of trade in decentralized economies– is a problem of Complexity that economic theory has not yet solved (although some economists believe otherwise¹).

Strong evidence of complex thinking is also found in Thomas Malthus (1798, 1815) and his “biological analogy” of population evolutionary processes, even inspiring Darwin (1859). In Malthus appear notions (among many) such as “ever-changing social environment”, “crisis point” and “relative overpopulation” in the struggle for existence, which fit very well with the biological and social system perspective postulated by Complexity.

Even Marx’s (1859) rejection of the idea of the existence of socio-economic systems that tend to some stable equilibrium - that is, towards the of history - coincides with the postulate of the theory of complex economic systems of “*socio-economies in eternal disequilibrium*” - that is, that typically do not reach any equilibrium (*steady state*).

Menger and Braunmüller (1871), although indiscriminately associated with the emergence of neoclassicism, laid the foundations of “methodological individualism”, stating that the central theme of economic theory was to understand the origin and emergence of spontaneous –i.e., non-deliberate– social structures from the interaction of individual agents. The study of the emergence of currency was one of the most important examples of the application of this view.

On the other hand, Marshall (1890, 1919), one of the founders of neoclassical thought in economics, affirmed that biology was the “Mecca” of economics, although in his work he did not follow this idea predominantly. Even so, in socioeconomics and economic policy, his main interest was poverty and how to reduce it, which would lead him to wonder about the evolution of the socioeconomic institutions that made it remain and reproduce in the same way. Thus, the human condition and its relationship with the environment were always among his concerns.

1 For examples of this, see Mas-Colell et al. (1995, p. 549), Arrow & Hahn (1971), Starr (1997), and more recently Rodrik (2015).

However, Marshall also had another marginal research front that would bring him closer to the theory of Complexity: the spatial dynamics of industrial systems (agglomerations, clusters, etc.), which would lead him to study their “industrial districts” to improve the collective functioning, instead of isolated companies in geographic space. These dynamics that Marshall applies here are known today in the literature as “*positive feedbacks*”, where each state is an “amplification” of the previous one, which is a substantial characteristic of (almost) every complex system, as will be described below.

Similarly, Veblen (1898, 1900), anthropologist, sociologist, psychologist, and economist, made a critical analysis of the institutions that determine people’s daily behavior (e.g., fashions, ceremonies, emulation, envy, rivalry, etc.), and how they evolve. In fact, he made a fundamental critique of the narrow neoclassical view of what a “market” was, asserting that one should investigate the *historical causal sequences of events*, as endogenous outcomes, and from there observe whether behavioral practices help (or not) to solve society’s perceived problems. There is no doubt that the evolutionary economics of Veblen and his followers (Commons, Ayres, Myrdal, Polanyi, Georgescu-Roegen, Kapp, Simon) is today a lively interdisciplinary paradigm in the social sciences with a strong influence on the complex vision of socioeconomics.

For his part, Keynes (1936) initial challenge to ensure convergence to full employment equilibrium in the long run, led him to see that capitalist economies were inherently dynamic and unstable; and that this was ignored by neoclassical theory, already in trouble in the Great Depression of the 1930s. Despite the subsequent efforts of Hicks (1937) and Samuelson (1947), and with the rational expectations theory of Muth (1961), to neutralize these problems with the Keynesian cross, they failed to adequately overcome the problem of aggregation that Keynes had already pointed out.

But not only this. Keynes revolutionized the Micro-Macro relationship in economics by posing that the problem of “composition” (or aggregation) was open: *the whole is more than the sum of its parts*. Moreover, deeply conversant with the problems of uncertainty (in fact, he published “A Treatise on Probability” in 1921), he asserted that there was no scientific basis on which to form any calculable probability. Therefore, the micro and macro behavior of agents influence each other; each one is the “foundation” of the other one.

At least for these (among other) fundamental elements –the problem of aggregation (better understood, as we shall see, by the concept of “non-ergodicity” of a complex system) and that of uncertainty, which after the later contributions of Shackle (1938) would be condensed in the concept of “fundamental uncertainty” – we can say that Keynes (and also the post-Keynesian school of Kalecki, Kaldor, Goodwin, Pasinetti, Minsky and Davidson) share with Complexity some similar views.

In order not to lengthen this very brief account with the contributions of many other economists, let us finally (but importantly) mention Joseph Schumpeter (1939), who with his theory of the “evolution of capitalist institutions”, his notion of “entrepreneur”, and his evolutionary mechanisms of selection, imitation, and innovation (Darwinian or not), have been, as well, a source of inspiration for the dynamics of the complex economies of a capitalist economy.

As can be seen in broad strokes, complexity theory in economics is a movement that does not gather totally new ideas (it is not a “new approach”), since many of them are ideas of economists of the last three centuries. But since the 1990s—in a germinal way through the Santa Fe Institute in New Mexico, USA (Fontana, 2010) and many other centers around the world—, it does seek to restore them, frame them and develop them theoretically with a holistic systemic vision, in such a way that they allow to understand the functioning of modern capitalist economies (with their high technological development, unexpected financial crises, bubbles, recessions, environmental problems -climate change-, poverty traps, segregation, etc.) and to apply the corresponding public policies, for which, as we have already said, it has become necessary to integrate economics with other social sciences.

Seven universal characteristics of a complex system

Some of the basic characteristics that help to define what a complex system is, but do not define it completely, are the following seven (Monsalve & Ávila, n. d.). These are, however, common in physics, biology, ecology, anthropology, medicine, engineering, information technology, among others. That is why this phenomenon called “*universality*” is key to the understanding of complex systems.

a. The system is made up of numerous heterogeneous units that interact. This is the “microeconomic base” of the system.

In an economic system viewed under complexity, these units can be human beings, companies, or the central government itself—which is considered here simply as one of the agents (units) of the system—.

b. The interaction of these units is governed by simple, albeit non-linear, adaptive rules. These are rules of the form “if these conditions are given... then this happens: ...”. It is these rules that form the “mesoeconomic basis” of the system.

In an economic system viewed under complexity, these are not high-level rules of cognitive introspection, but, rather, rules of inductive behavior. The reason for this is that, according to modern psychology, human beings in situations that are complicated or undefined are only moderately good at deductive logic and make only moderate use of it. However, we are outstanding in seeing or recognizing *patterns* that confer evolutionary benefits of survival and *well-being*. And for that, we construct internal temporal models or hypotheses (*schemata*) to work with (Arthur, 1994; Sauce & Matzel, 2017).

Human beings make localized deductions based on our current hypotheses and make decisions accordingly. As feedback comes in “from the outside”, we strengthen or weaken our beliefs in our current hypotheses and discard those that no longer work, replacing them with new ones.

In other words, when we cannot reason completely, or the problem is not well defined, we resort to simple models to fill in the gaps in our understanding. This is called *inductive behavior*.

For example, in chess players typically study the configuration of the board at a given time and recall their opponent's moves in past games (de Groot, 1965). They then use "adaptive rules" to form hypotheses or internal models about the opponent's possible responses to their possible moves. *But there is no total rationality here*. Typically, there is adaptation and non-Bayesian evolution; that is, the "ability" to survive and win. However, modeling all this requires different tools, especially computational ones. That is computer programs that can "learn" and recognize themselves, creating hypotheses, adapting, discarding, and mutating².

But also, for years, results from *behavioral* experiments, field experiments, intra-, and extra-species studies, archaeological and anthropological data, models of cultural evolution, and innovations in classical and evolutionary game theory have been integrated to show us a deep insight into the role of cooperative prosocial human behaviors (norms and institutions) in the creation and evolution of economies (Dhami, 2016).

c. The dynamics of the simple adaptive rules of the micro-units show, most commonly (but not exclusively), positive feedbacks.

These are iterative amplification (or reinforcement) mechanisms (each step is reinforced or augmented by the next) that occur in (almost) all complex systems: positive feedbacks amplify the current state of the system's behavior. Thus, for example, a behavior can become contagious and disperse in the system, but it can also disappear.

One of the best-known positive feedbacks is the "*conformity effect*", which is characterized by the property that a certain behavior of a micro-unit becomes more likely if "nearby" units have the same behavior. In turn, "*network effects*" ("*network externalities*" or *network effect*) are a particular case of conformity effect in which closeness is established by the distance (in ties) of one micro-unit to another one in a network (Figure 1).

Particularly, in an economic system viewed under complexity, financial panics and the formation of *ghettos* or similar forms of segregation are possible examples of the operation of these network effects. That is also the case in a market, where a good shows network effects if the value for a new buyer of adopting the good is increasing with the number of buyers who have already adopted it. Thus, the more customers adopt the good, the more valuable it becomes to potential adopters. However, these positive feedbacks can also work in reverse: if the adoption does not reach the "*critical mass*" of buyers, the good may eventually disappear from the market.

2 One of them is Netlogo software, a programmable multi-agent modeling environment.

Another very explanatory example in which positive *feedbacks* play a central role is Pólya's *stochastic urn processes* (Pólya, 1930). Here a process with conformity effects is formulated. Let us consider an urn of infinite capacity, to which are added balls of two colors: red and black. Suppose we start with a red and a black ball inside the urn, adding one ball at a time, indefinitely, according to the following rule: *we randomly choose a ball in the urn; if it is red, we add it to the urn, in addition to the extracted ball, another red ball; and if it is black, we add, similarly, an additional black ball.*

Then we can ask ourselves: does the proportion of red and black balls stabilize around a single ratio? That is, does the law of large numbers apply? Pólya (1930) showed that the proportion of red balls does tend to a limit X with probability 1, but that this is a random variable uniformly distributed between 0 and 1. X is a random variable uniformly distributed between 0 and 1. That is, structure (regularity) does emerge, but the realization of this structure is random: the regularity that emerges is not an equilibrium (*steady state*), but a random variable!

In the more general case, when the urn starts from an arbitrary number of red and black balls, the proportions, once again, tend to a distribution X which this time is a *two-parameter beta function*:

$$B(x,y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt \quad x,y > 0, \quad [1]$$

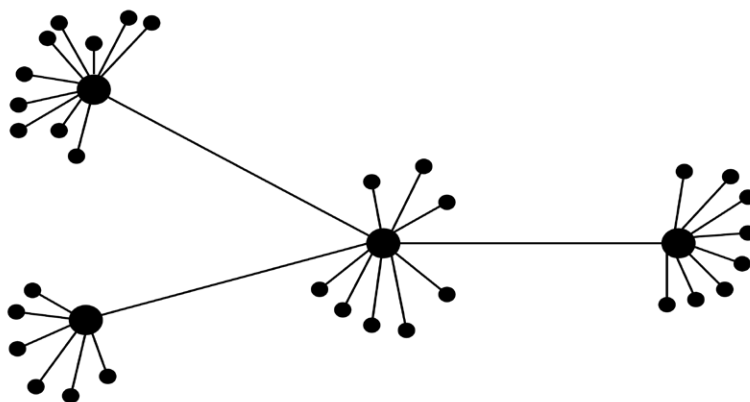
which is one of the best-known regularity-generating functions in the theory of complex systems. In fact, for a certain wide range of parameters (fixed y and relatively large x), the beta function generates *stochastic power laws* of the form $p(x) = Cx^{-\alpha}$ with $C, \alpha > 0$ fixed, which, by taking logarithms on both sides of the equation, is in the form of a decreasing straight line in log-log scale:

$$\log p(x) = \log C - \alpha \log x. \quad [2]$$

These Pólya urn models are very naturally used to represent phenomena in statistical mechanics, evolutionary biology, population processes and cases of emergence and formation of industrial clusters, and even disease contagion processes (Banerjee et al., 1999; Eggenberger & Pólya, 1923; Jhonson & Kotz, 1977).

A final example of the emergence of positive *feedbacks* is the *scale-free network* (Barabási et al., 1999; Jackson, 2019) in which the micro-units, as they enter the system, “adhere” to the units (nodes) that, at that moment, have more units already connected. These networks will be characterized, as an emerging regularity, by some very popular large connection centers (called *hubs*) and others (the vast majority) with very few adherents, following (by number of adherents) a *stochastic power law* (see Figure 1). Note that, once again, a random variable appears as a macro regularity, through positive feedbacks.

Figure 1. Scale-free network*



* The nodes most highlighted in black are the “larger” hubs because they have more connected adherents. The size distribution of the nodes follows a power law of the form [2] above.

Source: Monsalve and Ávila (n. d.).

Other examples of such positive feedbacks abound. These include *herding* behaviors, *mobs*, *peer effects*, etc. (Bikhchandani & Sharma, 2001; Jackson, 2019). It should be noted that these positive feedbacks are just one of the drivers that make the dynamics of a complex system always “multi-level” and that, at the systemic level, “the whole is more than the sum of its parts”. After all, these amplifications are at the heart of the information transmission processes between micro-units.

d. The non-linear mesoeconomic dynamics of complex systems co-create behavioral “regularities” in the long run (self-organization).

Not only do the micro-units contribute to the formation of macro regularities, but these, in turn, contribute to the behavior of the micro-units (retroactive behavior of the system), in “eternal disequilibrium”. Arthur (2009) and Arthur et al. (2020) point out this characteristic of complex systems in an important way, in the case of the problem of innovation and technological development of an economy. First, they claim that the “skeleton” of an economy is made up of its technologies, and that the rest of the economy, i.e., trade activities, flows of goods and services, and investment, only “wrap around” that skeleton, although integrally dependent on it. Thus, the economy does much more than readjust to technological changes: it is also from it, in fact, that technologies emerge.

Let us observe the systemic causality proposed by Arthur: technology creates the structure of the rest of the economy (which adapts to new changes), and this, in turn, leads to the creation of other technological innovations. This systemic causality (technology creating the

economy-economy creating technology) is not widely seen in mainstream economic literature. This phenomenon is known as *structural change in the economy*. And since these changes do not occur on a day-to-day basis, it is economic historians who are responsible for studying it. Here we do not see the introduction of new technologies as the origin of small adjustments and growth, but of changes in the structural composition of the economy itself.

In practice, a new technological change (which has been made possible by a combination of previous technologies and has outperformed its rivals in competition within the economy) is a call for the creation of new industries. However, it should be cautioned, as Arthur also points out, that the *process of technological change does not follow a strictly Darwinian type of evolution (natural selection and mutation)*, but a “combinatorial evolution” in the manner of cellular automata (Mitchell, 2019).

Thus, this process of technological change requires, each time, a new industrial organization; which, in turn, may cause new technical and social problems; and from there niches of opportunities are created for new technologies (institutions) to solve this within the economy. However, there is nothing inevitable, nothing predetermined in all the above. In fact, very different combinations, and arrangements (technological and institutional) can solve the technical and social problems posed by the new technology.

Which combination, ordering and path is chosen within the network of previous technologies is, in part, a question that depends on *small historical changes*. For example, the order in which problems are studied, the predilections and actions of the people in charge of the studies and decisions. In other words, according to Arthur, it is not possible to predict future results, but we can understand the mechanisms by which technological innovations are obtained. Thus, technology determines the structure of the economy, but which technology emerges is not determined *a priori*.

Today, economics is a discipline that is often criticized because, unlike the “hard sciences” such as physics or biology, it cannot be described by a set of facts that do not change over time (“economic laws”). But after the above discussion, it is postulated that this is not a flaw, but is proper and natural to it: *economics does not operate through “economic laws” in the manner of classical mechanics*. After all, it is not a simple system; the arguments seem to show it, rather, as a complex system in constant evolution, and the structures it forms are also constantly changing. This means that our interpretations of the economy must also change constantly over time.

e. In complex systems, micro-units behavior shows limited predictability of future behavior of the system...

Determinism, which is a doctrine of a necessary relationship between all events and phenomena, and conditioning to chance events, reached a development in natural science and materialistic philosophy with Bacon, Galileo, Descartes, Newton, Laplace, Spinoza, and the French materialists of the eighteenth century. Therefore, and in keeping with the level of the natural sciences at that time, determinism had a *mechanistic and abstract* character. Hence, an absolute value is

assigned to the form of causality. This is described according to the rigorously dynamic laws of classical mechanics, which leads to the identification of causality with necessity and to the denial of the objective character of chance.

The French mathematician Pierre Simon de Laplace (hence mechanical determinism is also known as Laplace's determinism) is the one who formulated this point of view with the greatest prominence. In 1799, Laplace began his five-volume *Traité de la Mécanique Céleste* with the statement that, *if the velocity and position of all the particles of the Universe were known at an instant, their past and future could be predicted.*

For more than 100 years Laplace's assertion seemed correct and, therefore, it was concluded that freedom would not exist, since everything was determined. However, the progress of science has refuted Laplace's determinism in all areas. Henri Poincaré's *chaos theory* (1903) and Werner Heisenberg's *uncertainty principle* (1927) were, at the time, two pillars in refuting the hypothesis of Laplacean determinism.

The origin of chaos theory arose in 1903 when Poincaré studied the old problem (*the three-body problem*) of whether the Earth-Moon-Sun system would be stable forever, and, surprisingly, found that it *depended sensibly on the initial conditions*. Specifically, the three-body problem consists in determining, at any instant, the positions and velocities of three bodies of any mass, subject to mutual gravitational attraction and starting from some given positions and velocities. Although it had already been rigorously studied by Newton, Euler, and Lagrange since 1687, Poincaré was the first to note the existence of an infinite number of periodic solutions with "high sensitivity" to the given initial positions and velocities.

Specifically, Poincaré noted that small variations in the initial conditions could imply large (in fact, *exponential*) differences in future behavior, thus making prediction impossible due to the problem of numerical approximation in those initial conditions. Note that this happens although these systems are strictly speaking deterministic (they are not random); that is, their behavior can be completely determined by knowing their initial conditions. From then on, examples of this "chaotic" behavior began to be observed not only in this problem of classical mechanics, but also in demographic, climatic, epidemiological, etc. problems.

On the other hand, the establishment of the *indeterminacy (or uncertainty) principle* by Werner Heisenberg (1927), in quantum mechanics, also revealed the inconsistency of Laplacean determinism by showing that quantum particles do not follow definite trajectories: *it is not always possible to know exactly the value of all the physical quantities that describe the particle's state of motion at any moment, but only a statistical distribution. Therefore, it is not always possible to assign a trajectory to a particle, although it is possible to say that there is a certain probability that the particle is in a certain region of space at a certain time.*

Chaos theory and Heisenberg's uncertainty principle (among others), led, then, to the conclusion that *science is not a predictive function through universal laws, but, very deeply, a function of understanding through relationships, patterns, or regularities of the objects of study.*

In the case of an economy, and despite Poincaré and Heineberg, the notion of uncertainty in a complex system is *not preponderantly established, today, around the notion of chaos*, as it is in other sciences. This is because since the 1990s, the tests developed for its detection in time series do not favor the hypothesis that the economic variables are chaotic (Faggini & Parziale, 2012), even though, curiously, the dynamics of Walras' *tâtonnement* and the dynamics of the spider's web (among many others), which are the basic dynamics with which microeconomics theoretically explains the formation of equilibrium prices, are chaotic (Kaizoji, 2010; Saari, 1995). That is why, currently, uncertainty in complex economic systems is being established, instead, around the notion of "fundamental uncertainty" à la Shackle (1938).

For Shackle (1938), each agent must form subjective beliefs about his and others' futures, and they will do the same. So, there is no such thing as an "optimal trajectory". For example, different entrepreneurs who are starting high-tech companies may not know how well their individual technologies will do, how the government will regulate them or what their competitors will be, and so on. This happens because they are subject to fundamental uncertainty and, thus, the problem they face is not well defined, in the manner indicated by the dominant neoclassical school.

Formally, Shackle objected to the representation of individual beliefs by an *additive measure of probability* à la Savage (1954), as with expected utility or expected benefit. The reason is that probabilities do not apply to decisions that are not repeated under the same conditions, nor it is possible to establish, *a priori*, the complete list of "states of nature." That is, in Shackle's world, decision-makers cannot divide an event into small "micro-events". Then, fundamental uncertainty ensures that even if agents have intelligent or sensible behaviors, they cannot have rational deductive behaviors because deductive rationality is simply not well defined.

Thus, the well-known "choice under rationality" is not well defined either: after all, there can be no logical solution to a problem that is not well defined. In other words, there is *no such thing as rational choice*. Moreover, as we said before, people normally act all the time in situations that are not well defined, forming hypotheses—or internal models—about the situation they are in, and continually updating them. In fact, people are adopting and discarding their hypotheses, strategies, and actions as they explore. That is, they always *act by induction rather than deduction*.

Something of extreme importance to mention here, and which has been seen in complex systems, is that it seems that the *inductive shaping of expectations* is a very strong contributor to the elimination of indeterminacy at the macro level, as shown by Arthur (1994) in the *El Farol* model of expected attendance at a restaurant, among many other examples in the same direction (Cara et al., 2000; Savit et al., 1999; Swain & Fagan, 2019). Although there is still much research to be done on this point, everything leads us to believe that proceeding by induction is at the heart of the behavior of an economy that is presented to us in permanent disruptive movement, as agents explore, learn, and adapt. An example of this can be seen in Ávila (2022), which presents an approach to the problem of the Invisible Hand posed by Adam Smith (1776/1969) that, inspired by the game of the Minority (Challet & Zhang, 1997;

Savit et al., 1997), by adding adaptive and/or evolutionary behaviors. In this paper, problems of segregation, poverty traps, bubbles –and therefore crises–, cyclicity, out-of-equilibrium dynamics, incomplete information, among many others, are observed; and these are standard characteristics of observable markets.

f. A complex system satisfies the non-ergodicity condition.

In words, the time averages of the behavioral trajectories of the micro-units of the system exist but cannot be obtained by means of the long-term averages in the aggregate. Thus, the statistical behaviors of any macro regularity of the system are not determined by the statistical behavior of the micro-units.

The definition of ergodicity is inspired by the famous *ergodic theorem* –George Birkhoff (1917, 1931)– which was a key piece in the study of *statistical mechanics* of systems (Gibbs, 1902). As this is a very general theorem (it requires measurement theory to be fully understood), it is common to assume that a system is ergodic if the average outcome (and its variance) of a large group of units coincides with the time-averaged outcome (and its variance) of a single unit over time. If these two outcomes coincide in mean and variance, the system is ergodic. For example, flipping a coin (heads or tails) is an ergodic stochastic process, because if a very large number N of people flip a coin, the average result (and its variance) will be the same as if only one person flipped the same coin N times.

Therefore, in a non-ergodic system, the means (and variances) of the individual states do not determine the mean (and variances) of the joint distribution of the system. Thus, if one wants to determine whether a stochastic process is (*a priori*) ergodic (or not), a first (and fundamental) test is to observe whether the following two outcomes are (or are not) equal: i. Average (and variance) of single-unit outcomes over time; ii. Average (and variance) of the results of a very large number of units at a single point in time. This shows the difference between individual (micro) average behavior and aggregate (macro) average behavior. Consequently, in a non-ergodic system the *law of large numbers is not satisfied*. After all, the law of large numbers is an application of *Birkhoff's ergodic theorem*.

In the same sense, and as the famous meteorologist Edward Lorenz (1993) would have pointed out, individual trajectories are analogous to day-to-day weather, while long-term averages are analogous to seasonal weather. Thus, in the case of weather, long-term averages reveal the full picture of the dynamic process with much more information than an individual trajectory could ever provide.

Going a little further, it can be shown from Birkhoff's ergodic theorem that a *non-ergodic* system has three additional fundamental characteristics:

- *Path-dependence*. This is immediate because the ergodicity condition implies that the time-averaged trajectory of any statistic is independent of the initial condition of the system. It is shown that the dynamic behavior of the system depends sensitively on the initial condition at each stage.
- Any *shock* to some part of the system at a certain stage affects the system in the long run. This is for the same reason of dependence on the average time path under the initial condition.
- The process from micro-units to long-term macro regularities is *irreversible*. This means that from a macro regularity, it is not always possible to predict exactly which were the micro-units that formed it, nor its dynamics.

When the system is non-ergodic, the interaction in any economy will be, then, by *hierarchical levels*, or, in colloquial terms, in the form of “terraces”. That is, at each level, the stochastic characteristics of the interactions are completely different. This, as we know, is known as the “hierarchical structure” of the complex system.

A famous example of positive feedbacks and path-dependence arising from a non-ergodic process in the high-tech industry is the QWERTY (patented by Christopher Scholes, 1868) versus the DVORAK (Dvorak et al., 1936) on the keyboards of old typewriters and today’s computers. This was studied by Paul David as early as 1985, who claimed that the adoption of the QWERTY keyboard is a consequence of decentralized and uncoordinated decisions in an environment where there are strong network effects. The latter, remember, means that the use of one standard or another will depend on how many others have adopted it as well, and this could lead to everyone choosing a single standard (which may not be the best).

These network externalities led to the QWERTY keyboard being “locked in” as a technological standard, even though there were more efficient and ergonomic keyboards such as the DVORAK. So, due to network effects, the market dominance of the QWERTY keyboard prevailed, even though it was only one of several long-term standards that could have emerged. And of course: under a different sequence of shock realizations at the beginning of the process, the DVORAK keyboard would have emerged as the standard keyboard.

Phenomena such as QWERTY are very common. This is also the case with the most widely used languages in the world and their preponderance (English, for example), which, despite not being the most grammatically rich, are typically used today in almost all Western countries and in many Eastern ones as well. It is also the well-studied case of the size and distance between rails of trains: the standard used today was not the best option.

On the other hand, let us emphasize that the relationship between ergodicity and chaos is not completely established. However, curiously enough, many of the chaotic systems to which the economic literature has resorted are ergodic, which has facilitated the analysis and the

obtaining of average results, at the cost of having this highly demanding hypothesis. Therefore, chaos, in general, can be attenuated (or, in some cases, regulated) if the system is ergodic. An example of this is the stochastic *cobweb* dynamics in the partial equilibrium model, which, in addition to being chaotic, Huang & Day (2001) proved that it is ergodic.

However, it is common to find multiple examples in economics in which the dynamics with positive *feedbacks* are non-ergodic, meaning that the expected value of changes in utility does not reflect the average growth over time. A simple example is that of a bet with a correct coin: *Heads*, gains 50% of the current wealth; and *Tails*, loses 40% of the current wealth. *W* and *Stamp* loses 40% of it. Counter-intuitively, while this gamble has an expected value of 1.05 times the current wealth per flip, then, the expected value of the change in utility is 1.05 times the current wealth per flip:

$$E = \frac{1}{2} W(150\%) + \frac{1}{2} W(60\%) = 105\%W, \quad [3]$$

also has a time average of, approximately, 0.95 times the current wealth, as this is shown (Peters & Gell-Mann, 2016) to be equal to:

$$(1-40\%)^{\frac{1}{2}} (1+50\%)^{\frac{1}{2}} W = 0.94868W \approx 0.95\% W. \quad [4]$$

Therefore, maximizing the expected value is wrong, and this person could eventually go bankrupt.

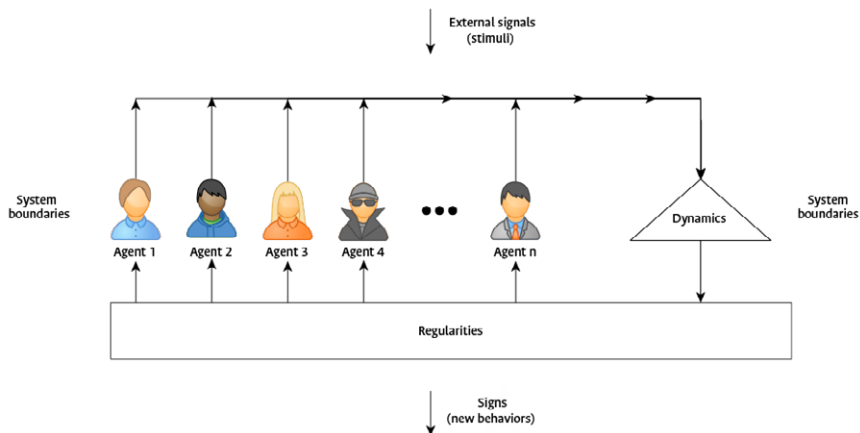
Ultimately, when making economic decisions about wealth, people often want to know how fast their personal fortunes will grow under different scenarios. But this requires determining what happens (over time) to wealth. Mistakenly assuming that wealth behavior is ergodic, replacing average temporal wealth with the expected value, can bankrupt individuals. It is now known that *wealth is not ergodic* (see, for example Peters & Gell-Mann, 2016), so the average behavior of a population cannot be deduced from averaging the temporal behavior of an individual.

This example shows that under non-ergodicity, temporal behavior implies that agents will have to adapt their preferences according to the dynamics and history (*path-dependent*) of the situations they face. *Non-ergodicity is a criticism at the heart of expected utility theory* (Poitras, 2013). Indeed, it is at the very origin of the typical textbook criticisms of Allais, Ellsberg, and the St. Petersburg Paradox. But, very importantly, it is at the heart of why the non-existence (except in very particular cases) of the representative agent, to which micro-founded macroeconomics appeals (Gorman, 1961; Jackson & Yariv, 2019).

g. A complex system adapts and evolves in response to the environment (system boundary) that surrounds it (Figure 2).

For Complexity, an economy must be seen as a socioeconomic system that develops multiple endogenous control mechanisms that make it function, but these mechanisms are continuously evolving (*evolving*) integrated with other complex boundary systems (system boundaries) such as, for example, the *biosphere*. After all, the biosphere is also a prototype of a complex system that includes the economies of all species and where the regularities of higher levels emerge from localized interactions and processes of adaptation, reaction and innovation carried out at lower levels (Capra & Luisi, 2014).

Figure 2. Diagram of the universal operation of a complex system



Source: Monsalve & Ávila (n. d.).

In fact, terrestrial life has survived multiple crises over millions of years, and the pandemic we are facing today is one of them: we are being threatened by a virus that has been generated in the biosphere. To survive, human organisms have, among others, a mechanism known as “*herd immunity*”. When a disease spreads among the population, an individual can protect himself by means of a vaccine that immunizes him. But this is not all because immunization of the individual means that he or she will never transmit the disease to others, effectively reducing the possibility of the disease proliferating in the population. Because of this, *a disease can be eradicated, even if the entire population has not been vaccinated*. This herd immunity effect is, of course, positive feedback from those we have already discussed, only now present in the interaction between complex systems (Solé & Elena, 2018).

Another problem we face, which is one of interaction between complex systems in the biosphere, is the threatening climate change, which manifests itself as a physical phenomenon, and originated (as a *start-up*) by the human economy in interaction with its frontiers. In 2018, William Nordhaus received the Nobel Prize for his work on global warming, where he claimed that allowing the Earth to warm by 3.2°C would provide the optimal *trade-off* between climate change and “damage” to the economy. But most scientists who are experts on the subject think that this level of warming would be disastrous. It is now known that the problems with Nordhaus’s analysis stem from an overestimation of the cost of combating climate change and an underestimation of the damage it would cause, as well as the use of an oversimplified model based on unrealistic assumptions.

Since some years ago, Complexity theory has been presenting a new perspective on this problem. For example, Farmer et al. (2020) have analyzed the costs of the transition to green energy and obtain results opposite to Nordhaus. For example, the data show that energy costs over the last century and a half (fossil fuel era) are surprisingly constant, and the *forecasts* show that renewables would lower energy costs substantially. So regardless of climate change, economically it is much better to convert to renewables, and doing so quickly would benefit us even more.

In fact, the authors show that it is possible to change most of the energy system within the next 20 years, save a lot of money and keep global warming below 2°C. Arriving at these results was the authors’ commitment to empirically grounded model building with the criteria of a complex economy (starting from the *bottom up*). Nordhaus, on the contrary, starts from aggregate models that rely on unrealistic assumptions such as rationality and utility maximization.

Complexity theory offers us, then, the possibility of using very modern tools to understand the economy (of human beings) and to suggest effective solutions. We are now close to the possibility of collecting data on economic activity at a remarkable level of detail and relying on powerful computers (*quantum computing*) to process them.

Complexity in financial systems

For several decades, traditional macroeconomists have lost the battle in the main arenas of discussion. Their place was taken by the advocates of the DSGE (*Dynamic Stochastic General Equilibrium*) model such as Thomas Sargent and Robert Lucas, which gave a boost to general equilibrium thinking in economics because it further entrenched this type of orthodox thinking.

Then came the 2008 financial crisis, which led to questioning the DSGE approach and wondering why its macroeconomics had failed to foresee the crisis. With this, the DSGE model showed its limitations and Complexity economics began to be considered as possible input for policy proposals. For example, at that time the President of the European Central Bank (Jean Claude Trichet) said:

Scientists have already developed sophisticated tools to analyze complex dynamical systems in a rigorous manner. These models have proven useful in understanding very important and difficult phenomena: epidemics, weather conditions, mass psychology, and magnetic fields. I am hopeful that central banks can also benefit from this and develop tools for analyzing financial markets and monetary policy transmission (Colander & Kupers, 2014, p. 171).

Indeed, this was the case. The theoretical works of, for example, Gabaix (2016) –among others– on stochastic power laws and their fat tails have been recognized by many economists as a satisfactory answer to what happened not only in that crisis, but also in other financial mini-crises and, even, in the Great Depression of 1929. Let us explain this a little more.

Let us start by saying that, from a computational point of view, the first complex model of asset markets was built at the Santa Fe Institute in New Mexico (USA) by Palmer et al. in 1994 (*The Santa Fe Artificial Stock Market*), and continued later by Palmer et al. in 1999. This model shows the effects of different agents when they have access to different sets of information and predictive behaviors.

Essentially, this is a heterogeneous agent version of the classical model of Robert Lucas (1978), in which heterogeneous agents (artificial investors) form a market inside the computer where a single type of asset is traded. Each investor monitors the price of the asset and submits bids that will determine the price of the asset the next day. To send the bids, the agents form different multiple hypotheses of what is moving the market, act according to the ones they consider best, and learn by creating new hypotheses and discarding the ones they think give bad results.

The authors of this computational model then placed an immense diversity of options for each agent and (against all expectations) *two different regimes began to emerge*:

- a. If agents updated their assumptions at a relatively slow rate, the diversity of expectations collapsed into a regime of homogeneous rational expectations. The reason was simple: if most investors believe something close to the rational expectations forecast, it becomes a “great attractor”; the others, not being close to rational expectations, will lose and thus slowly (since there is time) learn their way to that forecast. This is the regime predicted by the *Efficient Markets Hypothesis*, which is usually resorted to in asset market theory.
- b. But if the rate of hypothesis updating was relatively fast, the market was transitioning into a “complex regime,” with properties very similar to those seen in many real markets. In effect: what is seen in the computational model is the development of a rich “psychology” of divergent beliefs that do not converge over time. Very simple expectation rules, such as those made by agents in an ordinary asset market, appeared randomly in the population of all hypotheses and came to reinforce each other (positive feedbacks). Thus, in this regime, subpopulations of mutual feedback expectations emerged, which could then fall for the same reason. This is a recurrent phenomenon in asset markets: *certain types of expectations arise that reinforce each other, cause lock-in for a while and then disappear*.

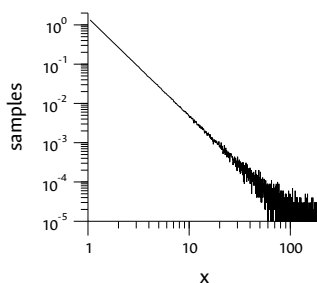
Another phenomenon also detected in this artificial market was the appearance of “avalanches” or “cascades” of individual behaviors (some small and some large) that changed aggregate behavior, which, in turn, induced behavioral changes at the individual level. This is the typical *out-of-equilibrium* behavior that occurred when agents changed their expectations (perhaps to explore some new ones), which disrupted the market somewhat, but could cause other agents to change their expectations as well, and “avalanches” of various measures were created, thus causing periods of high and low-price volatility. This is a phenomenon that is also seen in real financial market data, but not in DSGE equilibrium models.

What made this more interesting is that these “avalanches” or financial crises were shown to have properties associated with the thick tails of *stochastic power laws* (see Graph 3), where the size of the avalanche is inversely proportional to its frequency. This is as if the system were “self-regulating”, being then halfway between certainty and chaos (*edge of chaos*) (Bak & Chen, 1991), something very common in the behavior of the macro regularities of a complex system.

The characterization of this second regime of the artificial model is called the *Fractal Markets Hypothesis*, which shows that the behavior of the series of daily returns of an asset over a short period resembles its behavior over a long period (*self-similarity*), very similar to what happens recurrently in nature: for example, trees and their branches have fractal behavior because they are self-similar; and something very similar happens with the circulatory, nervous, lymphatic, etc., networks in mammals.³

Now, one implication of this hypothesis of fractal asset markets is that due to the presence there of thick tails (see Figure 3) in their power law expression, *there are many more extreme events (“black swans”) than would occur if the distribution were, for example, Gaussian (or normal), or were the result of a series of fixed probability events, such as coin flipping or dice rolling. In fact, stock market practice shows that in an asset market that includes about 1,000 assets, a standard deviation of 10 happens almost every day!*

Figure 3. Log-log scale stochastic power law, indicating the presence of a thick tail



Source: Newman (2005, p. 326).

3 Note that a stochastic power law of the form $p(x) = Cx^{-\alpha}$ has self-similarity behavior, since if $t > 0$ is a scalar, then one has that $p(tx) = t^{-\alpha}p(x)$. This feature of fractals is (almost) unique to this type of distribution.

Based on this, Gabaix (1999) and Gabaix & Ioannides (2004) showed that the thick tail in the power law of the distribution of firms in the financial market—which is called, in this case, “Gibrat’s law” (Gibrat, 1931)— could explain the regularities of large crashes in stock markets. However, this hypothesis is not only consistent with large crashes, but also with the overall distribution of small crashes described by the power law. In the same vein, the authors hypothesize that asset market crashes are due to a few very large financial institutions selling under pressure in illiquid markets (see also Solomon & Richmond, 2001).

This explanation, derived from power laws, begins by noting that large institutions are (almost) Gibrat distributed; so, when they trade, they could have a very large impact on prices through the “fat tail effect”. That is, when large financial institutions sell under pressure, they cause markets to fall and even crash.

Thus, according to this theory, the 2008 financial market crash was due to the movements of very large funds, with repercussions for the less large markets and for the bond markets. The “small crash” of 2010 would also have the same origins. There is even tentative evidence that a similar process developed at the beginning of the large financial market crashes of 1929 and 1987 (Gabaix et al., 2006; Kyle & Obizhaeva, 2016).

Let us note, finally, that power laws are also very quickly becoming central tools for analyzing wealth inequalities (Benhabib et al., 2011; Gabaix & Maggiori, 2015; Lucas & Moll, 2014; Piketty & Zucman, 2014; Toda & Walsh, 2015), among many other economic regularities.

Complexity in urban systems

Urbanization is one of the most complex processes facing the human species and its economies. Currently, although only 4% of the earth is urbanized and densely populated, just under 60% of the world’s population now lives in urban areas; this figure is expected to rise to 70% within 30 years. However, despite their importance as an economic factor, the ability to scientifically understand the processes of urbanization (and cities) and their impact on the biosphere has been limited, although there are currently many efforts to understand them scientifically. The great difficulties here lie in the fact that cities have many interdependent facets, social, economic, infrastructural, and spatial. They coexist within a spatial system that operates at different scales.

However, a growing and very interesting developing literature (Bettencourt, 2013a; West et al., 1997; West, 2017) asserts that all cities could evolve, not as organisms in the Darwinian sense, but according to certain basic principles operating at the local level in the ecological sense. From this perspective, West (2017) considers a city as a “social reactor”: that is, *not from the analogy with an organism but as an ecology, from its systemic dissipative function of energy.*

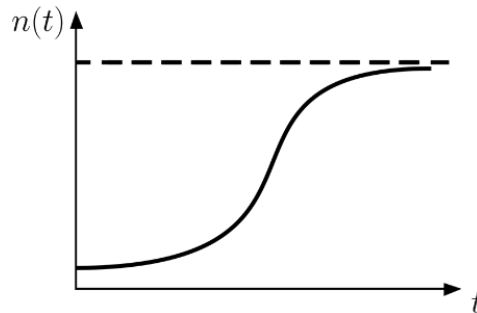
More specifically, this literature deals with certain theoretical principles that allow predicting average social, spatial, and infrastructural properties of cities, inspired by certain “metabolic scaling-laws”, originating from Kleiber’s (1932) law in mammals, which states that the metabolic rate (i.e., the daily energy requirement) of a mammal of mass M is $M^{3/4}$. Thus, in the same lifespan, a cat that has a mass $M = 100$ times that of a mouse ($M = 1$), will consume only $100^{3/4} = 32$ times the energy used by the mouse. Therefore, the larger the mammal, the less energy it will consume proportionally, and this type of “economy of scale” is apparently true for all physiological processes.

For example, it has been proven that, if the mass of a mammal is known, a scaling law can be used to describe physiological processes such as how much food it should eat in a day, what its heart rate per minute is, how long it will take to mature, its life horizon, how many hours a day it should sleep, etc. Even the efficiency of circulatory systems also follows a scaling law: doubling the average weight leads to a 25% increase in efficiency, and the mammal will live 25% longer (Brown et al., 2004).

Faced with all this, West et al. (1997) proposed a very profound (and controversial) explanation of this phenomenon: life does not operate in 3 dimensions but in four spatial dimensions (or five if time is included): three are the Euclidean dimensions of the organism, and the fourth is the “fractal dimension” which, in vague words, is the “size” of the fractal but resorting to a measure that generalizes the Euclidean one, and that even allows having fractional dimensions and not always positive integers as in the Euclidean (Mitchell, 2009). That is, according to this interpretation, the life of a mammal “takes advantage” of the possibility of using the interior space of the organism with systems (networks) that have fractal geometry since this type of network maximizes the transport and transfer of energy by taking resources from the environment. The explanation (in very brief words) is the following:

First, the authors state that organisms have evolved by natural selection paced by minimization of energy dissipation and maximization of *scaling* for energy attainment, transport, and transfer; and that this is very well fulfilled by scale-free networks. That is, life is sustained, to some extent, by scale-free networks. West et al. (1997) show, then, that, in the case of mammalian metabolism, the scaling law $M^{3/4}$ is of the form $M^{d/(d+1)}$ where $d = 3$ is the dimensionality of the space, and the 1 (one) is the increase due to the fractal dimension of the organism.

- c. Secondly, the authors also claim that the dynamics and fractal geometry of the networks control biological growth (*pace of life*) at all scales, leading to an emerging “universal” time scale: the organism grows systematically in the manner of a *sigmoid function* with a maximum lifetime t (Figure 4): at first, the organism grows slowly, during another period it grows very rapidly and then it returns to slow growth once again, until its death.

Figure 4. sigmoid function of biological growth

Source: West (2017, Chapter 10).

But this did not stop there. The work of West and other researchers (Bettencourt, 2013a; Brown et al., 2005; West et al., 1997; West, 2017) on metabolism in mammals made a seemingly unexpected turn toward *metabolic rates in urban centers*. After all, cities are made up of network systems that are nothing more than energy distribution structures and, as such, have their own “metabolism.”

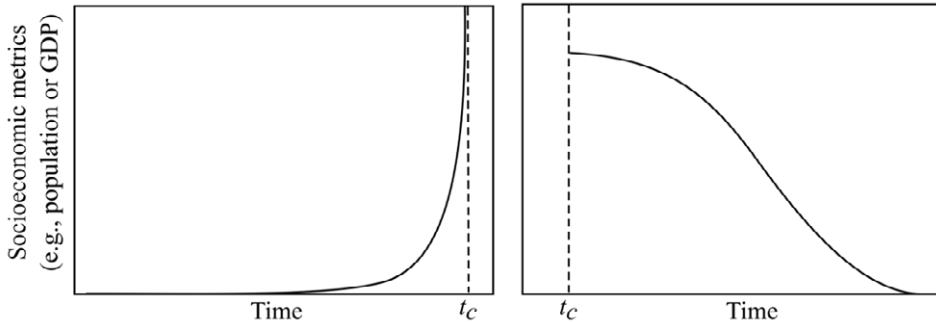
What West et al. (1997) initially found empirically is that, by doubling the size of a city, it will need (approximately) 15% less than twice as many roads, twice as much electrical wiring and twice as many gas stations, etc.; but it will also produce 15% more than twice the income, twice the wealth, twice the number of colleges and universities, twice the number of police officers, while having crime, disease and litter rates 15% more than twice as high. *This “economy of scale” behavior (15% rule, regardless of the city) was the beginning of a new theory for cities.*

More specifically, rates of social quantities such as wages or new inventions are observed to increase (in per-capita terms) with increasing city size, in a “superlinear” manner with scale $b = 1 + d > 1$ as $d \approx 0.15$. However, the rates of urban infrastructure quantities (roads, power, and water cabling, etc. -all per capita-) decrease sublinearly with scale $b = 1 - d < 1$ for the same $d \approx 0.15$. These data show something by now very familiar: that larger cities are not only more costly and congested but also more creative than small towns.

Empirical results such as these have been pointing out that one of the most important systems of an economy, such as the cities, is a relatively simple socioeconomic phenomenon: its average global properties can be described by a few key parameters (West, 2017). In fact, these (and other) regularities have been confirmed in thousands of cities around the world, only having data from a few urban systems with different levels of development. That is, some socioeconomic measures are independent of city size and, therefore, could be useful as means to evaluate urban planning policies. This is the case, for example, of land use, urban infrastructure, and socioeconomic activity rates.

The origin of these behaviors of scale, and explanations of how spatial, infrastructural, and social interdependencies take place, have also begun to be understood, although much remains unknown (Bettencourt, 2013b; Bettencourt & Lobo, 2019; West, 2017). For example, an explanation of why cities do not behave precisely as biological organisms, but as ecosystems (West, 2017), can be found in Figure 5.

Figure 5. Ecosystem behavior of a city without innovation.

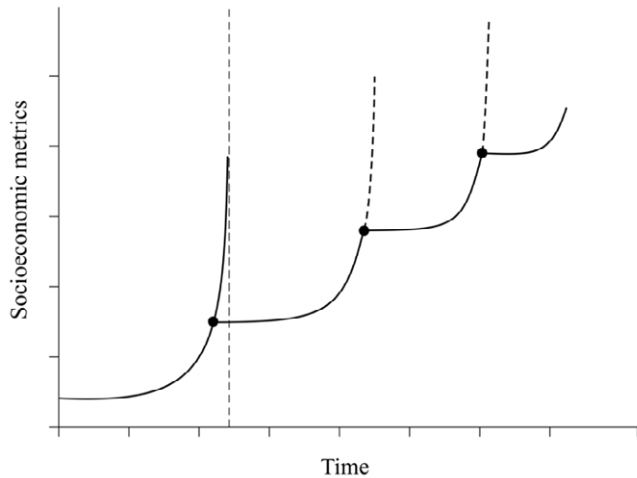


Source: West (2017, Chapter 10).

There it is illustrated that the dynamics and geometry of the networks that control the growth rate of a city make its progression take place in an increasing manner but approaching a singularity (dotted vertical straight line) in finite time, and not in the manner of a sigmoid function—as in the case of biological organisms, for example, a person or any other mammal—as shown in Figure 5. Then West (2017) explains that upon reaching the singularity (in finite time), *the city goes to collapse (due to exhausted capacities) unless systematic innovations are carried out and occur more and faster*, as suggested in Figure 6.

What is happening here, then, is a continuing tension between innovation and wealth creation versus growth at the scale of the city economy. Note that the reaction times to each period with innovation are getting shorter and shorter, so it is feared that the acceleration seen over the last 150 years (telephone, television, computers, Internet, cell phones, etc.) is a sign that the final reckoning of the world’s largest cities has begun.

On the other hand, West (2017) would also apply his groundbreaking work on biological systems and cities to the world of firms. This research led him to show *that the growth of firms, unlike cities, behaves sigmoidally in the manner of biological organisms*. Companies will grow in a sigmoidal fashion, but due to excess energy dissipation by growing solely based on technological advantages (increasing returns to scale), with a lot of bureaucracy and no innovation, they will be doomed to disappear ... just as will happen to us, individual biological organisms.

Figure 6. Illustration of the ecosystem behavior of a city with innovation

Source: West (2017, Chapter 10).

In the end, all the empirical results in this section suggest that there are deep and relatively simple interdependencies between the spatial, infrastructural, and social facets of a city. So, it is clear today that to understand an economy it is fundamental to understand the behavior within and between the urban centers in which we live, both economically and socially. That is, how much information about an economy is transmitted from the internal behavior of the cities.

Brief conclusions

The differences between the theory of complex economic systems and other economic currents or schools of thought are obvious: Complexity suggests that an economy is not a mechanical system but a complex (organic) system; that is, it is always adapting and evolving. It does not seek general results that occur at any time (“laws”), nor does it seek equilibrium (*steady states*), nor does it assume rationality of agents in the *mainstream way*. Instead, it seeks relatively simple macro regularities that develop spontaneously and change over time through interaction with itself and with other systems: complex systems are always in adaptive and evolutionary motion, expanding, and allowing new temporal regularities to emerge.

Thus, to analyze these systems, simple sets of adaptive and evolutionary rules are sought (computationally or theoretically) to explain the emergence of a certain regularity that arises empirically from the data, instead of establishing a set of equations that determine those improbable static equilibria (*steady states*). For example (as a non-exhaustive illustration), from

theory there is already a very well identified set of “generating functions” (one of them is the beta function, mentioned above) that produce many of the most typical regularities that have appeared so far in economics (for example, fractals expressed under a stochastic power law). As already from computational methods, *artificial intelligence* is considered a fundamental tool to explain many of the emerging economic phenomena. However, it must be recognized that the work is just beginning.

In the end, complexity theory as a holistic science knows that, in the face of the great difficulty of economic problems, the best option is to have a “highly educated economic judgment”. In other words, it is to go at a cautious and prudent pace, with no immediate pretensions of becoming an “economic school”: in fact, most scholars in the theory of complex economies barely consider it a “movement”. So, the main message that this systemic view of economics, with its broad vision, methods, and tools, sends us is that the best “economic education” we can ever have is that of “educated common sense” since there will never be any perfect recipe or theory for the difficult circumstances we face on a daily basis. Finally, educated common sense is also ignorance, but perhaps at a lesser level.

References

- [1] Arrow, K., & Hahn, F. (1971). *General Competitive Analysis*. Holden-Day.
- [2] Arthur, W. (1994). Inductive Reasoning and Bounded Rationality. *The American Economic Review*, 84(2), 406-411. <https://www.jstor.org/stable/2117868>
- [3] Arthur, W. (2009). *The Nature of Technology: What It Is and How It Evolves*. Free Press.
- [4] Arthur, W., Beinhocker, E., & Stanger, A. (2020). *Complexity Economics: Dialogues of the Applied Complexity Network. Proceedings of the Santa Fe Institute's 2019 Fall Symposium*. SFI Press. Santa Fe Institute.
- [5] Ávila, D. (2022). *Spin-glass y la Mano Invisible de Adam Smith* [master's degree thesis, Universidad Nacional de Colombia - Sede Bogotá]. <https://repositorio.unal.edu.co/handle/unal/82914>
- [6] Bak, P., & Chen, K. (1991). Self-Organized Criticality. *Scientific American*, 264(1), 46-53. <http://www.jstor.org/stable/24936753>
- [7] Banerjee, A., Burlina, P., & Alajaji, F. (1999). Image Segmentation and Labeling Using The Polya Urn Model. *IEEE Transactions on Image Processing*, 8(9), 1243-1253. <https://doi.org/10.1109/83.784436>
- [8] Barabási, A., Albert, R., & Jeong, H. (1999). Mean-Field Theory for Scale-Free Random Networks. *Physica A: Statistical Mechanics and Its Applications*, 272(1-2), 173-187. [https://doi.org/10.1016/S0378-4371\(99\)00291-5](https://doi.org/10.1016/S0378-4371(99)00291-5)
- [9] Bassett, D., & Claveau, F. (2018). El entomólogo económico: entrevista con Alan Kirman. *Revista de Economía Institucional*, 21(40), 343-366. <https://doi.org/10.18601/01245996.v21n40.13>
- [10] Benhabib, J., Bisin, A., & Zhu, S. (2011). The Distribution of Wealth and Fiscal Policy in Economies with Finitely Lived Agents. *Econometrica*, 79(1), 123-157. <https://doi.org/10.3982/ECTA8416>

- [11] Bettencourt, L. (2013a). The Origins of Scaling In Cities. *Science*, 340(6139), 1438–1441. <https://doi.org/10.1126/science.1235823>
- [12] Bettencourt, L. (2013b). Complexity, Cities and Energy. *International Seminar on Nuclear War and Planetary Emergencies – 45th Session*, 313–325. https://doi.org/10.1142/9789814531788_0027
- [13] Bettencourt, L., & Lobo, J. (2019). Quantitative Methods for The Comparative Analysis of Cities in History. *Frontiers in Digital Humanities*, 6. <https://doi.org/10.3389/fdigh.2019.00017>
- [14] Bikhchandani, S., & Sharma, S. (2001). Herd Behavior in Financial Markets. *IMF Staff Papers*, 47(3), 279–310. <https://www.imf.org/External/Pubs/FT/staffp/2001/01/pdf/Bikhchan.pdf>
- [15] Birkhoff, G. (1917). Dynamical Systems with Two Degrees of Freedom. *Transactions of the American Mathematical Society*, 18(2), 199–300. <https://doi.org/10.2307/1988861>
- [16] Birkhoff, G. (1931). Proof of The Ergodic Theorem. *Proceedings of the National Academy of Sciences*, 17(12). <https://doi.org/10.1073/pnas.17.2.656>
- [17] Brown, J., Gillooly, J., Allen, A., Savage, V., & West, G. (2004). Toward a Metabolic Theory of Ecology. *Ecology*, 85(7), 1771–1789. <https://doi.org/10.1890/03-9000>
- [18] Brown, J., West, G., & Enquist, B. (2005). Yes, West, Brown and Enquist's Model of Allometric Scaling is Both Mathematically Correct and Biologically Relevant. *Functional Ecology*, 19(4), 735–738. <https://doi.org/10.1111/j.1365-2435.2005.01022.x>
- [19] Capra, F., & Luisi, P. (2014). *The Systems View of Life*. Cambridge University Press. <https://doi.org/10.1017/CBO9780511895555>
- [20] Cara, M., Pla, O., & Guinea, F. (2000). Learning, Competition and Cooperation in Simple Games. *The European Physical Journal B - Condensed Matter and Complex Systems*, 13(3), 413–416. <https://doi.org/10.1007/s100510050051>
- [21] Challet, D., & Zhang, Y. (1997). Emergence of Cooperation and Organization in An Evolutionary Game. *Physica A: Statistical Mechanics and Its Applications*, 246(3–4), 407–418. [https://doi.org/10.1016/S0378-4371\(97\)00419-6](https://doi.org/10.1016/S0378-4371(97)00419-6)
- [22] Colander, D., & Kupers, R. (2014). *Complexity and The Art of Public Policy Solving society's problems from the bottom up*. Princeton University Press.
- [23] Darwin, C. (1859). *On the Origin of Species By Means of Natural Selection, or The Preservation of Favoured Races in The Struggle for Life*. John Murray.
- [24] David, P. (1985). Clio and The Economics of QWERTY. *The American Economic Review*, 75(2), 332–337.
- [25] de Groot, A. (1965). *Thought and Choice in Chess*. Mouton.
- [26] Deichmann, U. (2017). Hierarchy, Determinism, and Specificity in Theories of Development and Evolution. *History and Philosophy of the Life Sciences*, 39(4), 33. <https://doi.org/10.1007/s40656-017-0160-3>
- [27] Dhami, S. (2016). *The Foundations of Behavioral Economic Analysis*. Oxford University Press.
- [28] Dvorak, A., Merrick, N., Dealey, W., & Ford, G. (1936). *Typewriting Behavior: Psychology Applied to Teaching and Learning Typewriting*. American Book Company.
- [29] Eggenberger, F., & Pólya, G. (1923). Über die statistik verketteter vorgänge. *ZAMM - Zeitschrift Für Angewandte Mathematik Und Mechanik*, 3(4), 279–289. <https://doi.org/10.1002/zamm.19230030407>

- [30] Faggini, M., & Parziale, A. (2012). The Failure of Economic Theory. Lessons from Chaos Theory. *Modern Economy*, 03(01), 1–10. <https://doi.org/10.4236/me.2012.31001>
- [31] Farmer, J., Way, R., & Mealy, P. (2020). *Estimating the Costs of Energy Transition Scenarios Using Probabilistic Forecasting Methods*. Institute for New Economic Thinking. https://www.inet.ox.ac.uk/files/energy_transition_cost_INET_working_paper_with_SI1.pdf
- [32] Fontana, M. (2010). The Santa Fe Perspective on Economics: Emerging Patterns in The Science of Complexity. *History of Economic Ideas*, 18(2), 167–196. <https://www.jstor.org/stable/23723516>
- [33] Gabaix, X. (1999). Zipf's Law for Cities: An Explanation. *The Quarterly Journal of Economics*, 114(3), 739–767.
- [34] Gabaix, X. (2016). Power Laws in Economics: An Introduction. *Journal of Economic Perspectives*, 30(1), 185–206. <https://doi.org/10.1257/jep.30.1.185>
- [35] Gabaix, X., Gopikrishnan, P., Plerou, V., & Stanley, H. (2006). Institutional Investors and Stock Market Volatility. *The Quarterly Journal of Economics*, 121(2), 461–504. <https://doi.org/10.1162/qjec.2006.121.2.461>
- [36] Gabaix, X., & Ioannides, Y. (2004). The Evolution of City Size Distributions. *Handbook of regional and urban economics*, vol. 4, (pp. 2341–2378). Elsevier. [https://doi.org/10.1016/S1574-0080\(04\)80010-5](https://doi.org/10.1016/S1574-0080(04)80010-5)
- [37] Gabaix, X., & Maggiori, M. (2015). International Liquidity and exchange Rate Dynamics. *The Quarterly Journal of Economics*, 130(3), 1369–1420. <https://doi.org/10.1093/qje/qjv016>
- [38] Gibbs, J. (1902). *Elementary Principles in Statistical Mechanics: Developed with Especial Reference to The Rational Foundation of Thermodynamics*. C. Scribner's Sons.
- [39] Gibrat, R. (1931). *Les inégalités économiques*. Recueil Sirey.
- [40] Gorman, W. (1961). On a Class of Preference Fields. *Metroeconomica*, 13(2), 53–56. <https://doi.org/https://doi.org/10.1111/j.1467-999X.1961.tb00819.x>
- [41] Heisenberg, W. (1927). Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. *Zeitschrift Für Physik*, 43(3–4), 172–198. <https://doi.org/10.1007/BF01397280>
- [42] Hicks, J. (1937). Mr. Keynes and the “Classics”; A Suggested Interpretation. *Econometrica*, 5(2), 147–159. <https://doi.org/10.2307/1907242>
- [43] Huang, W., & Day, R. (2001). On the Statistical Properties of Ergodic Economic Systems. *Discrete Dynamics in Nature and Society*, 6(3), 181–189. https://www.emis.de/journals/HOA/DDNS/Volume6_3/189.pdf
- [44] Jackson, M. (2019). *The human network: how your social position determines your power, beliefs, and behaviors*. Vintage.
- [45] Jackson, M., & Yariv, L. (2019). *The Non-Existence of Representative Agents* [working paper]. <https://doi.org/10.2139/ssrn.2684776>
- [46] Jhonson, N., & Kotz, S. (1977). *Urn Models and Their Application*. John Wiley & Sons.
- [47] Kaizoji, T. (2010). Multiple Equilibria and Chaos in A Discrete Tâtonnement Process. *Journal of Economic Behavior & Organization*, 76(3), 597–599. <https://doi.org/10.1016/j.jebo.2010.09.008>
- [48] Keynes, J. (1921). *A treatise on probability* (Vol. 31, Issue 2). Dover Publications.
- [49] Keynes, J. (1936). *The General Theory of Employment Interest and Money*. Macmillan and Co.

- [50] Kleiber, M. (1932). Body Size and Metabolism. *Hilgardia*, 6(11), 315–353. <https://hilgardia.ucanr.edu/Abstract/?a=hilg.v06n11p315>
- [51] Krugman, P. (1996). *What Economists Can Learn from Evolutionary Theorists* [speech]. European Association for Evolutionary Political Economy. <http://www.mit.edu/~krugman/evolute.html>
- [52] Kyle, A., & Obizhaeva, A. (2016). Market Microstructure Invariance: Empirical Hypotheses. *Econometrica*, 84(4), 1345–1404. <https://doi.org/10.3982/ECTA10486>
- [53] Lorenz, E. (1993). *The Essence of Chaos*. UCL.
- [54] Lucas, R. (1978). Asset Prices in An Exchange Economy. *Econometrica*, 46(6), 1429–1445. <https://doi.org/10.2307/1913837>
- [55] Lucas, R., & Moll, B. (2014). Knowledge Growth and The Allocation of Time. *Journal of Political Economy*, 122(1), 1–51. <https://doi.org/10.1086/674363>
- [56] Malthus, T. (1798). *An Essay on the Principle of Population*. McMaster University Archive for the History of Economic Thought.
- [57] Malthus, T. (1815). *An Inquiry into The Nature and Progress of Rent, and The Principles by Which It Is Regulated*. <https://www.gutenberg.org/files/4336/4336-h/4336-h.htm>
- [58] Marshall, A. (1890). *Principles of Economics*. Macmillan.
- [59] Marshall, A. (1919). Industry and trade. *The Journal of Education*, 89 (20), 544–545. <https://doi.org/10.1177/002205741908902008>
- [60] Marx, K. (1859). *Zur Kritik der politischen Oekonomie*. Henricus.
- [61] Mas-Colell, A., Whinston, M., & Green, J. (1995). *Microeconomic Theory*. Oxford University Press.
- [62] Menger, C., & Braumüller, W. (1871). *Grundsätze der volkswirtschaftslehre*.
- [63] Mitchell, M. (2009). *Complexity: A Guided Tour*. Oxford University Press.
- [64] Mitchell, M. (2019). *Artificial Intelligence: A Guide for Thinking Humans*. Penguin UK.
- [65] Monsalve, S., & Ávila, D. (n. d.). *Microeconomía y Complejidad* [working paper].
- [66] Muth, J. (1961). Rational Expectations and The Theory of Price Movements. *Econometrica*, 29(3), 315–335. <https://doi.org/10.2307/1909635>
- [67] Newman, M. (2005). Power laws, Pareto distributions and Zipf’s law. *Contemporary Physics*, 46(5), 323–351. <https://doi.org/10.1080/00107510500052444>
- [68] Palmer, R., Arthur, W., Holland, J., & LeBaron, B. (1999). An artificial stock market. *Artificial Life and Robotics*, 3(1), 27–31. <https://doi.org/10.1007/BF02481484>
- [69] Palmer, Richard, Arthur, W., Holland, J., LeBaron, B., & Tayler, P. (1994). Artificial economic life: a simple model of a stockmarket. *Physica D: Nonlinear Phenomena*, 75(1–3), 264–274. [https://doi.org/10.1016/0167-2789\(94\)90287-9](https://doi.org/10.1016/0167-2789(94)90287-9)
- [70] Peters, O., & Gell-Mann, M. (2016). Evaluating Gambles Using Dynamics. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 26(2), 023103. <https://doi.org/10.1063/1.4940236>
- [71] Piketty, T., & Zucman, G. (2014). Capital Is Back: Wealth-Income Ratios in Rich Countries 1700–2010. *The Quarterly Journal of Economics*, 129(3), 1255–1310. <https://doi.org/10.1093/qje/qju018>
- [72] Poincaré, H. (1903). L’espace et ses trois dimensions. *Revue de Métaphysique et de Morale*, 11(3), 281–301. <http://www.jstor.org/stable/40892786>

- [73] Poitras, G. (2013). *Ergodicity and the history of neoclassical economic theory* [working paper]. https://www.sfu.ca/~poitras/HES_erg.pdf
- [74] Pólya, G. (1930). Sur quelques points de la théorie des probabilités. *Annales de l'institut Henri Poincaré*, 1(2), 117–161. www.numdam.org/item/AIHP_1930__1_2_117_0
- [75] Prigogine, I., & Stengers, I. (1997). *The End of Certainty: Time's Flow and The Laws of nature*. Free Press.
- [76] Rodrik, D. (2015). *Economic Rules : Why Economic Works, When It Fails, and How to Tell The Difference*. Oxford University Press.
- [77] Saari, D. (1995). A Chaotic Exploration of Aggregation Paradoxes. *SIAM Review*, 37(1), 37–52. <https://doi.org/10.1137/1037002>
- [78] Samuelson, P. (1947). *Foundations of Economic Analysis*. Harvard University Press.
- [79] Sauce, B., & Matzel, L. (2017). Inductive reasoning. In J., Vonk, & T., Shackelford (eds) *Encyclopedia of Animal Cognition and Behavior*, 6 (1-8). Springer. https://doi.org/10.1007/978-3-319-47829-6_1045-1
- [80] Savage, L. (1954). *The Foundations of Statistical Inference*. John Wiley & Sons.
- [81] Savit, R., Manuca, R., & Riolo, R. (1997). *Adaptive Competition, Market Efficiency, Phase Transitions and Spin-Glasses*. <http://arxiv.org/abs/adap-org/9712006>
- [82] Savit, R., Manuca, R., & Riolo, R. (1999). Adaptive Competition, Market Efficiency, and Phase Transitions. *Physical Review Letters*, 82(10), 2203–2206. <https://doi.org/10.1103/PhysRevLett.82.2203>
- [83] Schumpeter, J. (1939). *Business Cycles: A Theoretical, Historical and Statistical Analysis of The Capitalist Process* (vol. 1). McGraw-Hill.
- [84] Sen, A., Paul, F., & Stiglitz, J. (2010). *Mismeasuring our lives: Why GDP doesn't add up*. The New Press.
- [85] Shackle, G. (1938). *Expectations, Investment and Income*. Oxford University Press.
- [86] Smith, A. (1969). *An inquiry into the nature and causes of the wealth of nations*. W. Strachan and T. Cadell (original published on 1776).
- [87] Solé, R., & Elena, S. (2018). *Viruses as complex adaptive systems*. Princeton University Press.
- [88] Solomon, S., & Richmond, P. (2001). Power Laws of Wealth, Market Order Volumes and Market Returns. *Physica A: Statistical Mechanics and Its Applications*, 299(1–2), 188–197. [https://doi.org/10.1016/S0378-4371\(01\)00295-3](https://doi.org/10.1016/S0378-4371(01)00295-3)
- [89] Starr, R. M. (1997). *General Equilibrium Theory : An Introduction*. Cambridge University Press.
- [90] Swain, A., & Fagan, W. (2019). Group Size and decision Making: Experimental Evidence for Minority Games in Fish Behaviour. *Animal Behaviour*, 155, 9–19. <https://doi.org/10.1016/j.anbehav.2019.05.017>
- [91] Toda, A., & Walsh, K. (2015). The Double Power Law in Consumption and Implications for Testing Euler Equations. *Journal of Political Economy*, 123(5), 1177–1200. <https://doi.org/10.1086/682729>
- [92] Veblen, T. (1898). Why Is Economics Not an Evolutionary Science? *The Quarterly Journal of Economics*, 12(4), 373–397. <https://doi.org/10.2307/1882952>
- [93] Veblen, T. (1900). The Preconceptions of Economic Science. *The Quarterly Journal of Economics*, 14(2), 240–269. <https://doi.org/10.2307/1883770>
- [94] West, G. (2017). *Scale: The Universal Laws of Life, Growth, and Death in Organisms, Cities, and Companies*. Penguin Press.
- [95] West, G., Brown, J., & Enquist, B. (1997). A General Model for The Origin of Allometric Scaling Laws in Biology. *Science*, 276(5309), 122–126. <https://doi.org/10.1126/science.276.5309.122>