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Modeling of Modified Value-At-Risk for the Skewed Student-T Distribution

Sukono[1]
[1]Department of Mathematics Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, 45363 Jatinangor, Indonesia
sukono@unpad.ac.id

Wahyuddin[2]
[2]Faculty of Economics and Business, Universitas Malikussaleh, Lhokseumawe, Aceh, Indonesia
wahyuddin@unsika.ac.id

Nunung Nurhasanah[3]
[3]Faculty of Economics and Business, Universitas Singaperbangsa, Karawang, 41362 Jawa Barat, Indonesia
nunung.nurhasanah@unsika.ac.id

Jumadil Saputra[4]*
[4]School of Social and Economic Development, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia
jumadil.saputra@umt.edu.my

Yuyun Hidayat[5]
[5]Department of Statistics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, 45363 Jatinangor, Indonesia
yuyun.hidayat@unpad.ac.id

Abstract

This paper discusses the modeling of Modified Value-at-Risk (MVaR) for asset returns of skewed Student-T distribution. MVaR for skewed Student-T distribution is a special form of MVaR

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models of nonnormal distribution. As a result, this model can be used to determine the amount of market risk. Student-T distribution is, especially used for asset returns. In conclusion, the performance of each model Value-at-Risk applied in accordance with the distribution of stock returns is quite good. It is shown that the values of QPS are in the interval [0, 2], and tend to be close to zero.

Keywords: Skewed, Distribution, Mclaurin, Gram-Charlier.

Modelización del valor en riesgo modificado para la distribución sesgada de Student-T

Resumen

Este documento analiza el modelado del Valor en riesgo modificado (MVaR, por sus siglas en inglés) para los retornos de activos de la distribución sesgada de Student-T. MVaR para la distribución sesgada de Student-T es una forma especial de los modelos MVaR de distribución no normal. Como resultado, este modelo se puede utilizar para determinar la cantidad de riesgo de mercado. La distribución de Student-T se utiliza especialmente para la devolución de activos. En conclusión, el rendimiento de cada modelo de Valor en Riesgo aplicado de acuerdo con la distribución de los rendimientos de las acciones es bastante bueno. Se muestra que los valores de QPS están en el intervalo [0, 2] y tienden a estar cerca de cero.

Palabras clave: sesgado, distribución, Mclaurin, Gram-Charlier.

1. INTRODUCTION

The Value-at-Risk (VaR) is a measure of market risk done by using a standard normal distribution approach, with assumed that the asset returns unilabiate normal distribution, has two parameters mean and standard deviation. The issue is how to determine the VaR estimate quartile to the normal distribution when given the level of significance. The modeling of MVaR in this paper is done by breaking the probability density function skewed Student-T distribution using a Mclaurin series with a differential operator. Based on Mclaurin series of a differential operator, Gram-Charlier series a function that is expressed to the probability density distribution skewed Student-T was then formed. MVaR models have been formulated to determine the amount of market risk, particularly skewed Student-T distribution for asset returns. As a numerical illustration, some of the return of shares traded in the stock market in Indonesia was analysed.

2. LITERATURE REVIEW

This section discusses the modelling of modified Value-at-Risk for the skewed Student-T distribution. However, as the basis for modelling the Value-at-Risk of the standard normal distribution approach is discussed in advance as follows.

2.1. Value-at-Risk of Standard Normal Distribution Approach

The standard method assumes that asset returns bivariate normal distribution has two parameters: the mean μ and standard deviation σ. The issue of VaR estimation is how to determine the percentile to α of the standard normal distribution z_{α} :

$$\alpha = \int_{-\infty}^{q} f(r)dr = \int_{-\infty}^{z_\alpha} \Phi(z)dz = N(z_\alpha) \text{, quantile}$$

$$q = z_\alpha \sigma + \mu \tag{1}$$

where $\Phi(z)$ is the density function of the standard normal distribution, $N(z)$ is the cumulative function of the standard normal distribution, r is the random variable of portfolio return, $f(r)$ is the normal distribution density function to return (log returns) with the mean μ and standard deviation σ, and q is the log returns the smallest if given confidence level α (Dokov et al., 2007). VaR estimation is done by the equation:

$$VaR = -W_0 \times q = -W_0(z_\alpha \sigma + \mu) \tag{2}$$

Where W_0 is the initial investment, and the minus sign (-) reported a loss (losses).

2.2. Value-at-Risk of Standard Student-T Distribution Approach

If it is assumed that the return r is the standard Student-T distributed with degrees of freedom is v, then the quantile of this distribution is $q = \mu + t_v^*(\alpha)\sigma$, where $t_v^*(\alpha)$ is the quantile to α of standard Student-T distribution with degrees of freedom $v = T - 1$, and T the number of data observations. The relationship between the quantile of standard Student-T distribution with degrees of freedom is v, denoted by t_v, and the standard distribution denoted by t_v^*, is:

$$p = P(t_v \leq q) = P\left(\frac{t_v}{\sqrt{v/(v-2)}} \leq \frac{q}{\sqrt{v/(v-2)}} \right) = P\left(t_v^* \leq \frac{q}{\sqrt{v/(v-2)}} \right)$$

Where $v > 2$. Namely, if the quantile to α of standard Student-T distribution with degrees of freedom v, then $q/\sqrt{v/(v-2)}$ is quantile to α of standard Student-T distribution (Tsay, 2005). Therefore, if given probability α and an initial investment of W_0, the Value-at-Risk (VaR) can be calculated using the equation:

$$VaR = -W_0 \times q = -W_0 \left(\mu + \frac{t_v(\alpha)\sigma}{\sqrt{v/(v-2)}} \right) \qquad (3)$$

Where $t_v(\alpha)$ is the quantile to α of standard Student-T distribution with degrees of freedom $v = T - 1$, and T the number of data observations, and assuming a negative value to a α smallest?

2.3. Modified Value-at-Risk of Standard Normal Distribution Approach

Cornish-Fisher expansion is used to determine the percentile of the distribution of non-normal distribution. Cornish-Fisher expansion is intended to provide an adjustment factor to the estimated percentile of the distribution of non-normality, and the adjustment of the given normality is small. Therefore, Cornish-Fisher expansion can be used to the estimation of the VaR whenever Profit / Loss (P / L) has a distribution of non-normality. Suppose that z_α is the percentile of the standard normal distribution for the confidence level of α (for example $z_{0,05} = -1.645$ and so on). The meaning of Cornish-Fisher expansion is:

$$z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S + \frac{1}{12}(z_\alpha^3 - 3z_\alpha)K - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)S^2 + \textit{higher order terms} \tag{4}$$

Where S is the skewness and K is the kurtosis of a distribution. If we eliminate higher order terms because it is assumed the smaller influence of normality, the expansion becomes:

$$z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S + \frac{1}{12}(z_\alpha^3 - 3z_\alpha)K - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)S^2 \tag{5}$$

To use the expansion, we see the value of percentile z_α of the standard normal probability distribution table. This is equivalent to adjusting the normal percentile z_α for skewness and or kurtosis. The existence of non-normality of the asset that can be used as a guide for choosing the portfolio is different with regard to assumptions. According to Dowd (2004) and Linsmeier and Pearson (1996), Cornish-Fisher in 1937 developed a new measure where the risk is measured by standard deviation, skewness (for return asymmetry) and kurtosis (for return fat tails). The measure of this risk is called Modified Value-at-Risk (MVaR), which is a Value-at-Risk similar to a classic Value-at-Risk approach. According to Benninga and Wiener (1998), MVaR is formulated as:

$$MVaR = -W_0 \left\{ \mu + \left(z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)K - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)S^3 \right) \sigma \right\}$$

Where μ is the mean, σ is the standard deviation, S is the skewness, K is the kurtosis, and z_α is the percentile of the standard normal distribution with a significance level of α .

2.4. Back Test for Performance Evaluation of Vary

According to Dowd (2004), the back test method can be performed using the predicted outcome evaluation approach (forecast) introduced by Lopez in 1998. Suppose r_t is the rate of loss (if negative) or benefit (if positive) generated in the period t, and VaR_t predicted VaR for the period, under Lopez-II approach indicator, the function loss observation period t is given as follows:

$$C_t = \begin{cases} 1 + (r_t - VaR_t)^2; & \text{if } r_t > VaR_t \\ 0; & \text{if } r_t \le VaR_t \end{cases} . \quad (7)$$

To test the null hypothesis that the VaR_t model is the best, a function of quadratic probability score (QPS) can be given by:

$$QPS = \frac{2}{n}\sum_{i=1}^{n}(C_t - p)^2 , \tag{8}$$

Where p is the probability value that determined equal to the significance level (typically 5%). The QPS value lies between the range [0, 2], and the value of the QPS approach to zero, is the best model (Dowd, 2004). Referring to equation (22), the range [0, 2] shows that the number 0 is the minimum value which occurs when the entire $r_t \leq VaR_t$, and number 2 is the maximum value that occurs when the entire $r_t > VaR_t$.

3. METHODOLOGY

The modelling of Modified Value-at-Risk (MVaR) for Skewed Student-T distribution is an attempt to formulate and implement an investment risk measurement tool, especially on stock returns which follows the distribution of Skewed Student-T (non-normal). Simply put, MVaR models for skewed Student-T distribution is an attempt to describe a phenomenon in stock returns in the form of a mathematical formula that is easy to learn and do calculations. The modelling stages of MVaR for Skewed Student-T distribution is as given in Figure 1 below.

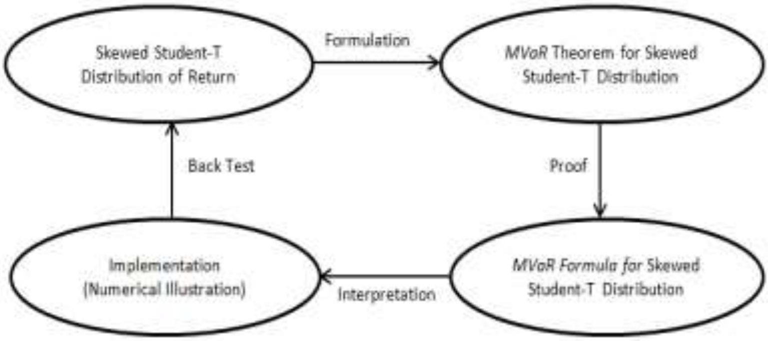

Figure 1: Modelling Stages

Figure 1 explained are (i) Identify and name random variables of stock returns that do not follow the Normal distribution or person who takes the Skewed Student-T distribution, and makes assumptions as necessary to simplify the phenomenon so that it can be traced mathematically (Kosari, 2018). (ii) Applying known mathematical theories to mathematical models that have been formulated to obtain mathematical conclusions in the form of the MVaR for Skewed Student-T distribution theorem. (iii) Way to prove the theorem formulated to draw mathematical conclusions in the form of the MVaR for Skewed Student-T distribution theorem, and interpret it as information relating to the problem being modeled by giving an explanation or making an estimate. (iv) Test estimates for real data, which in this study were tested using the Back Test based on the Lopez-II approach. If the estimates we make are not in line with reality, then the model that is obtained needs to be refined or formulate a new model and start recycling. It could also improve the assumptions given (Indriastuti, 2019).

4. MODELLING RESULT

In this section, we developed a theorem of Modified Value-at-Risk (MVaR) for an asset that follows the distribution of skewed Student-T. For a random variable that follows the distribution of skewed Student-T, referring to Hu and Kercheval (2010), Gaivoronski and Pflug (2005), Ciccio and Monti (2011), the probability density function can be expressed as:

$$g(r \mid v, \lambda) = b \frac{\Gamma\{(v+1)/2\}}{\sqrt{\pi(v-2)}\Gamma(v/2)} \left(1 + \frac{\xi^2}{v-2}\right)^{-(v+1)/2}$$

Where,

$$\xi = \begin{cases} (br+a)/(1-\lambda) \text{ jika } r < -a/b \\ (br+a)/(1+\lambda) \text{ jika } r \geq -a/b \end{cases}.$$

The constant a and b is defined as $a = 4\lambda c \dfrac{v-2}{v-1}$ and $b^2 = 1 + 3\lambda^2 - a^2$, and for the random variable r with mean zero and variance one, constants c is $c = \Gamma\{(v+1)/2\} / \sqrt{\pi(v-2)}\Gamma(v/2)$, with degrees of freedom $v = T - 1$ where T is the number of observation data. In relation to the formulation of a Modified Value-at-Risk (MVaR) model, this paper develops the theorem of MVaR Student-T distribution, as follows: Theorem 1. An asset returns when having skewed Student-T distribution, with degrees of freedom v, then MVaR of the initial capital W_0 and the significance level of α, can be calculated using the following equation:

$$MVaR = -W_0 \Big\{ \mu + [t_v(\alpha) + \tfrac{1}{6}\{t_v^2(\alpha) - 1\}S + \tfrac{1}{24}\{t_v^3(\alpha) - 3t_v(\alpha)\}K - \tfrac{1}{36}\{2t_v^3(\alpha) - 5t_v(\alpha)\}S^2] \frac{\sigma}{\sqrt{v/(v-2)}} \Big\}$$

where μ is the mean, S is the skewness, K is the kurtosis, and σ is the standard deviation, and $t_v(\alpha)$ is the percentile of α standard Student-T distribution with degrees of freedom $v = T - 1$, and T is the number of data observations, and $v > 4$.

Proof: Suppose r is a random variable return (log returns) assets. Furthermore, the standard Student-T distributed random variables have a probability density function as follows:

$$f(r \mid v) = c \left(1 + \frac{r^2}{v-2} \right)^{-(v+1)/2} \quad ; \; -\infty < r < \infty \qquad (10)$$

Where $c = \Gamma\{(v+1)/2\} / \sqrt{\pi(v-2)}\Gamma(v/2)$, and v is the degree of freedom.

According to Johnson and Kotz (1970), if $f(r \mid v)$ is the probability density function with a cumulant κ_1, κ_2, …, then the function:

$$g(r \mid v) = \exp\left[\sum_{j=1}^{\infty} \varepsilon_j \{(-D)^j / j!\}\right] f(r \mid v) \tag{11}$$

Will have a cumulant $\kappa_1 + \varepsilon_1$, $\kappa_2 + \varepsilon_2$, …. It is important to explain the equation (11). Operator

$$\exp\left[\sum_{j=1}^{\infty} \varepsilon_j \{(-D)^j / j!\}\right],$$

Has the general shape description Mclaurin series as:

$$\sum_{i=0}^{\infty}\left[\sum_{j=1}^{\infty} \{(-D)^j / j!\}\right]^i / i!$$

Suppose D expresses differential operator, and $D^j f(r) = d^j f(r) / dr^j$. For equation (9), differential order j from $f(r \mid v)$ expresses $D^j f(r \mid v) = P_j(r \mid v) f(r \mid v)$ [(1)].

---

(1) Example: $$P_1(r \mid v) = -2\left(\frac{v+1}{2}\right)\frac{r}{(v-2)}\left(1+\frac{r^2}{v-2}\right)^{-1}$$

$$P_2(r \mid v) = +4\left(\frac{v+1}{2}\right)^2 \frac{r^2}{(v-2)^2}\left(1+\frac{r^2}{v-2}\right)^{-2} - 2\left(\frac{v+1}{2}\right)\frac{1}{(v-2)}\left(1+\frac{r^2}{v-2}\right)^{-1}$$

$$+4\left(\frac{v+1}{2}\right)\frac{r^2}{(v-2)^2}\left(1+\frac{r^2}{v-2}\right)^{-2}$$

Referring to Chateau and Dufresne (2017), if $f(r|v)$ is the probability density function, then the Gram-Charlier series of $g(r|v)$ expressed in $f(r|v)$ is given as:

$$g(r|v) = f(r|v) - \varepsilon_1 Df(r|v) + \tfrac{1}{2}(\varepsilon_1^2 + \varepsilon_2)D^2 f(r|v) - \tfrac{1}{6}(\varepsilon_1^3 + 3\varepsilon_1\varepsilon_2$$

$$+\varepsilon_3)D^3 f(r|v) + \tfrac{1}{24}(\varepsilon_1^4 + 6\varepsilon_1^2\varepsilon_2 + 4\varepsilon_1\varepsilon_3 + \varepsilon_4)D^4 f(r|v) + \ldots, \quad (12)$$

It forms the cumulative distribution function of the equation (12) as:

$$\int_{-\infty}^{r} g(t|v)dt = \int_{-\infty}^{r} f(t|v)dt - \varepsilon_1 f(r|v) + \tfrac{1}{2}(\varepsilon_1^2 + \varepsilon_2)Df(r|v) - \ldots. \quad (13)$$

It should be noted again that the Cornish-Fisher expansion is intended to provide an adjustment factor to the estimated percentile of non-standard distribution. Thus, the expectation and standard deviation in the calculation of Value-at-Risk are unchanged (fixed). Because $D^j f(r|v) = P_j(r|v) f(r|v)$ and if the expected value and standard deviation of $f(r|v)$ and $g(r|v)$ is equal, then $\varepsilon_1 = \varepsilon_2 = 0$ (2) and $g(r|v)$ equation (10) become:

$$g(r|v) = [1 - \tfrac{1}{6}\varepsilon_3 P_3(r|v) + \tfrac{1}{24}\varepsilon_4 P_4(r|v) - \ldots] f(r|v), \quad (14)$$

---

(2) $$\int_{-\infty}^{\infty} rg(r|v)dr = \int_{-\infty}^{\infty} rf(r|v)dr - \varepsilon_1 \int_{-\infty}^{\infty} rDf(r|v)dr + \tfrac{1}{2}(\varepsilon_1^2 + \varepsilon_2)\int_{-\infty}^{\infty} rD^2 f(r|v)dr$$

$$\mu_g = \mu_f - \varepsilon_1 \int_{-\infty}^{\infty} rDf(r|v)dr + \tfrac{1}{2}(\varepsilon_1^2 + \varepsilon_2)\int_{-\infty}^{\infty} rD^2 f(r|v)dr$$

If $\mu_g = \mu_f$, he must $\varepsilon_1 = \varepsilon_2 = 0$. The same way, for verification $\varepsilon_1 = \varepsilon_2 = 0$ through $\sigma_g = \sigma_f$.

and the cumulative distribution function of the equation (14) is:

$$\int_{-\infty}^{r} g(t\,|\,v)dt = \int_{-\infty}^{r} f(t\,|\,v)dt - [\tfrac{1}{6}\varepsilon_3 P_2(r\,|\,v) - \tfrac{1}{24}\varepsilon_4 P_3(r\,|\,v) + ...] f(r\,|\,v) .$$

It is assumed that $P_j(r\,|\,v)\, f(r\,|\,v) \rightarrow 0$ is for moments of high degree (greater 4) of r. Generally it will be obtained from:

$$\int_{-\infty}^{r} g(t\,|\,v)dt = [\textstyle\sum_{i=0}^{\infty} D^{-1}\{\sum_{j=1}^{\infty} \varepsilon_j (-D)^j / j!\}^i / i!] f(r\,|\,v) . \text{ [3] } \quad (16)$$

For the Cornish-Fisher expansion, taken any value R_α and T_α such as that:

$$\int_{-\infty}^{R_\alpha} g(r\,|\,v)dr = \alpha = \int_{-\infty}^{T_\alpha} f(r\,|\,v)dr . \quad (17)$$

Whereas α is the probability value. Based on the equation (17) transformation functions may be formed:

$$R_\alpha = f(T_\alpha\,|\,v) = a_0 + a_1 H_1(T_\alpha\,|\,v) + a_2 H_2(T_\alpha\,|\,v) + a_3 H_3(T_\alpha\,|\,v) + ...$$

or,

$$T_\alpha = g(R_\alpha\,|\,v) = b_0 + b_1 H_1(R_\alpha\,|\,v) + b_2 H_2(R_\alpha\,|\,v) + b_3 H_3(R_\alpha\,|\,v) + ...$$

Where $H_j(x) = D^j f(x)$ determine the term to j of polynomial expansion.

---

[3] If $f(x)$ the probability density function, then the function

$$g(x) = \exp[\textstyle\sum_{j=1}^{\infty} \varepsilon_j \{(-D)^j / j!\}] f(x) = \{\sum_{i=0}^{\infty} [\sum_{j=1}^{\infty} \varepsilon_j \{(-D)^j / j!\}]^i / i!\} f(x)$$

Referring to Coi and Sweetman (2010), a non-standard process can be expressed by a polynomial expansion of the series as a standard process Hermetic:

$$R_\alpha = \lambda \left\{ T_\alpha + \sum_{j=1}^{\infty} h_j \mathrm{He}_{j-1}(T_\alpha) \right\} \tag{18}$$

$$\approx \lambda \left\{ T_\alpha + h_3(T_\alpha^2 - 1) + h_4(T_\alpha^3 - 3T_\alpha) + h_5(2T_\alpha^3 - 5T_\alpha) \right\}.$$

(19)

In the same way, a standard process can be expressed in non-standard processes as:

$$T_\alpha = R_\alpha - \sum_{j=3}^{\infty} h_j \mathrm{He}_{j-1}(R_\alpha) \tag{20}$$

$$\approx \lambda \left\{ R_\alpha + h_3(R_\alpha^2 - 1) + h_4(R_\alpha^3 - 3R_\alpha) + h_5(2R_\alpha^3 - 5R_\alpha) \right\}.$$

(21)

Where λ is the scaling factor that ensures R_α is have variance 1. He_j Polynomial Hermete to j, which $\mathrm{He}_1(\xi) = \xi$, $\mathrm{He}_2(\xi) = \xi^2 - 1$, $\mathrm{He}_3(\xi) = \xi^3 - 3\xi$, $\mathrm{He}_4(\xi) = 2\xi^3 - 5\xi$, with ξ any value.

Furthermore, because the Cornish-Fisher expansion aims to adjust the percentile of the distribution of non-standard to standard distribution, in this case only equation (21) is considered. Suppose κ_3 and κ_4 successively express skewness and kurtosis, the value h_j, $j=1,...,5$ successively is: $h_1 = h_2 = 0$, $h_3 = \kappa_3 / 6$, $h_4 = \kappa_4 / 24$ and $h_5 = \kappa_3^2 / 36$. When the chosen value $\lambda = 1$, and also substituting values h_j, $j=1,...,5$ into the equation (19) the following formula will be obtained:

$$R_\alpha = \left\{ T_\alpha + \tfrac{1}{6}(T_\alpha^2 - 1)\kappa_3 + \tfrac{1}{24}(T_\alpha^3 - 3T_\alpha)\kappa_4 - \tfrac{1}{36}(2T_\alpha^3 - 5T_\alpha)\kappa_3^2 \right\}. \qquad (22)$$

Based the equation (22), as well as referring to equation (3) and (5), Modified Value-at-Risk for the return of assets following the skewed Student-T distribution can be calculated using the following formula:

$$MVaR = -W_0 \left\{ \mu + [t_v(\alpha) + \tfrac{1}{6}\{t_v^2(\alpha) - 1\}S + \tfrac{1}{24}\{t_v^3(\alpha) - 3t_v(\alpha)\}K \right.$$

$$\left. - \tfrac{1}{36}\{2t_v^3(\alpha) - 5t_v(\alpha)\}S^2] \frac{\sigma}{\sqrt{v/(v-2)}} \right. . \qquad (23)$$

Where μ is the mean, σ is the standard deviation, S is the skewness (for the return of non-symmetry), and K is the kurtosis (for the return of the fat-tail), and $t_v(\alpha)$ is the quantile to α standard Student-T distribution and $v > 2$. □

5. NUMERICAL ILLUSTRATION

To provide an idea of the application of the models discussed in section 2, this section gives a numerical illustration. Numerical illustration begins from the analyzed data, estimated distribution model, calculation of Value-at-Risk, and backtesting to look at the performance of the model used, like the following (Yang et al., 2019).

5.1. Data

Stock data analyzed accessed through website http://www.finance.go.id//. Data consisted of 10 (ten) stocks are chosen, for during the period January 2, 2014 to June 4, 2017, which includes shares: INDF, DEWA, AALI, LSIP, ASII, TURB, HDMT, BMRI, UNTR, and BBRI. Furthermore, respectively they are called until. Data stock prices are then determined to return each by using a log stock return. Return data for stocks until will then be used to calculate the Value-at-Risk (VaR) or Modified Value-at-Risk (MVaR) with a parametric approach as follows (Soo et al., 2019).

5.2. The VaR Calculation by Parametric Approach

The calculation of VaR or VaR return stock with a parametric approach will be based on the standard normal distribution approach and standard Student-T distribution approaches. This is done because there are some stock returns that are a follow a normal distribution, and there is also a follow standard Student-T distribution. This way is expected to generate VaR calculations in accordance with the distribution. The steps are as follows: First, shaping of the distribution estimated returns for each stock S_1 until S_{10}. Based on the shape of the distribution estimator will estimate values of the estimators of mean $\hat{\mu}$, standard deviation $\hat{\sigma}$, skewness $\hat{\varsigma}$ and kurtosis $\hat{\kappa}$, for each stock returns. Second, determining the level of significance α which will be used, so the percentile can be determined to α appropriate distribution of the approach used, from the available distribution table (standard normal distribution or standard Student-T distribution). Using percentile to α and assuming the amount of the initial investment W_0, we calculated the VaR or MVaR. Third, back testing each VaR calculation results or MVaR. First, estimating the shape of the distribution of returns for each stock S_1 until S_{10}. The estimation is done using statistics of quantile-to-quantile plot (QQ-plot), and also using the Anderson-Darling (AD) statistics. The estimation is done using the software Minitab 14. One example is the estimation of the distribution of stock returns S_1. Histogram of stock returns S_1 is given in Figure 2 below (Sears, 2018).

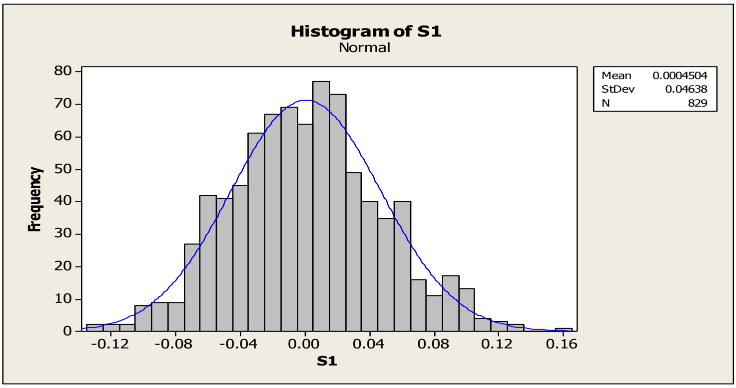

Figure 2: Histogram Stock Return of S_1

Figure 2, looks like a histogram of the normal distribution. Based on the histogram analysis, set hypotheses H_0 : stock returns S_1 follow a normal distribution, with alternative H_1 : stock returns S_1 do not follow a normal distribution. Hypothesis testing for distribution of stock returns estimator form S_1 performed using QQ-plots and statistics of AD. QQ-plot to test hypotheses stock returns S_1 is given in Figure 3 below.

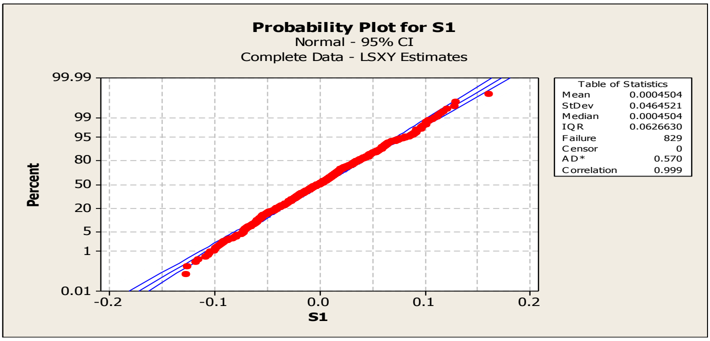

Figure 3: Distribution of Normal QQ-plot Stock Return of S_1

It appears that Figure 3 shows that the dots are formed almost entirely in a straight line from the chart, so that it can be concluded that the hypothesis H_0 can be accepted. It means that the stocks return data S_1 is an appropriate follow of a normal distribution. It is also supported by obtaining the statistical values of $AD = 0.570$ which is relatively small. Based on the estimator forms of distribution, it can estimate the parameters mean $\hat{\mu}$, standard deviation $\hat{\sigma}$, skewness $\hat{\varsigma}$ and kurtosis $\hat{\kappa}$, the results are given in the Table 1 (Matsubara & Yoshida, 2018).

Table 1: Descriptive Statistics of S1 Stock Returns

Variable	Count	Mean	StDev	Variance	Skewness	Kurtosis
S1	829	0.000450	0.04638	0.00215	0.16	2.91

Each parameter estimator is $\hat{\mu}_1$=0.000450, $\hat{\sigma}_1$= 0.04638, $\hat{\varsigma}_1$=0.160 and $\hat{\kappa}_1$=2.910. Estimates of distribution forms and parameter values are also carried out on nine other stock returns. Shape estimation results and the distribution of stock returns parameters S_1 until S_{10} are given in Table 2. Second, setting the level of significance α=0.05 of the standard normal distribution table obtained percentile $z_{0.05}$=-1.645. Here is assumed that the amount of the initial investment of $W_0 = 1$ rupiah (IDR) unit. Because stock returns of S_1 is a normal distribution, to calculate (VaR parameters require the mean $\hat{\mu}$ and standard deviation $\hat{\sigma}$. By using the mean $\hat{\mu}_1$, satandard deviation $\hat{\sigma}_1$, initial investment W_0 and percentile $z_{0,05}$, The VaR which is calculated using equation (2) is as follows:

$$VaR_1 = 1(0.000450 - 1.645 \times 0.04638 = 0.075845$$

The VaR calculation was thereby also conducted on other stocks return in accordance with the form of distribution. If the stock returns have the form of standard Student-T distribution, the VaR is calculated using the formula (3). If stock returns are not normally distributed, and did not follow the Student-t distribution, the VaR is calculated using the formula (6). Whereas, if following the Skewed Student-T distribution, the VaR is calculated using the formula (8). The VaR calculation results of the stock return S_1 until

S_{10} are summarized in Table 2. Third, back testing results of VaR calculations stock returns S_1 using Lopez-II approach, with the loss of function indicators (7). If probability value of $p = 5\%$, then using equation (15) QPS =0.0113728 is obtained. The calculation results for stock return S_1 until S_{10} are summarized in Table 2.

Stocks (S_i)	**Dist**	**Stat. AD**	**Mean ($\hat{\mu}_i$)**	**Std. Dev ($\hat{\sigma}_i$)**	**Skewness ($\hat{\varsigma}_i$)**	**Kurtosis ($\hat{\kappa}_i$)**	**VaR**	**QPS**
S_1	Normal	0.570	0.00045	0.04638	.160	2.910	0.07585	0.11373
S_2	Skew Student-T	8.280	0.00287	0.06487	.868	5.041	0.08692	0.15131
S_3	Logistic	8.199	0.00158	0.03680	0.030	5.880	0.09344	0.04203
S_4	Logistic	21.708	0.00027	0.03675	0.600	9.150	0.08843	0.05072
S_5	Logistic	3.597	0.00097	0.03429	.070	5.050	0.05492	0.08567
S_6	Skew Student-T	20.271	0.00151	0.05504	.193	11.032	0.05943	0.12559
S_7	Normal	0.312	0.00029	0.05302	.040	2.810	0.08692	0.00769
S_8	Logistic	3.529	0.00085	0.03371	.430	3.640	0.04789	0.12266
S_9	Logistic	7.265	0.00131	0.03765	.150	5.930	0.05450	0.11629
S_{10}	Logistic	1.764	0.00096	0.03337	.310	2.540	0.04922	0.10520
							Average	**0.09209**

From Table 2, it is shown that stock returns S_1, with probability $p = 5\%$ produces a value of QPS amount 0.113728. Value-at-Risk (VaR) of stock return S_1 showed performance either when the value QPS small near zero. Thus, VaR rate of return S_1 is a good enough performance. Therefore, it can be used as a measure of risk based on parametric approach. Similarly, VaR for the return of other stocks showed a good enough performance, because the value QPS is small enough based on parametric approach. Return stock S_1 until S_{10} that produces value QPS which is relatively small are respectively the stock return S_7 amount 0.007690; S_3 amount 0.042026; S_4 amount 0.050717 and S_5 amount 0.085666. Return of stocks that produces a value QPS which is relatively large are respectively S_2 amount 0.151306; S_6 amount 0.125594; S_8 amount 0.122664; S_9 amount 0.116292; S_1 amount 0.113728 and S_{10} amount 0.105205. However, all values of QPS each stock returns S_1 until S_{10} are still in the range [0, 2] and tend to be closer to zero value. Overall, the average value of stock returns S_1 until S_{10} amount is 0.092089.

6. CONCLUSION

This paper has discussed the modelling of the Modified Value-at-Risk for skewed Student-T distribution. The modelling is based on Cornish-Fisher expansion. This paper also discussed the Value-at-Risk (VaR) approach to the standard normal distribution, VaR approach to the standard Student-T distribution, and Modified Value-at-Risk (MVaR). Measuring the performance of the model of Value-at-Risk. It has performed by using back testing with Lopez-II approach. As a numeric illustration, this paper used ten (10) stocks notated as S_1 until S_{10}. In accordance with the distribution model of each analyzed stock, the amount of Value-at-Risk was then calculated, and back testing was also performed. Back testing results show that the performance of each model Value-at-Risk applied in accordance with the distribution of stock returns is quite good. It is shown that the values of QPS are in the interval [0, 2], and tend to be close to zero.

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