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A naïve approach to speed up portfolio optimization problem using a multiobjective genetic algorithm

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ABSTRACT

Genetic algorithms (GAs) are appropriate when investors have the objective of obtaining mean-variance (VaR) efficient frontier as minimising VaR leads to non-convex and non-differential risk-return optimisation problems. However GAs are a time-consuming optimisation technique. In this paper, we propose to use a naïve approach consisting of using samples split by quartile of risk to obtain complete efficient frontiers in a reasonable computation time. Our results show that using reduced problems which only consider a quartile of the assets allow us to explore the efficient frontier for a large range of risk values. In particular, the third quartile allows us to obtain efficient frontiers from the 1.8% to 2.5% level of VaR quickly, while that of the first quartile of assets is from 1% to 1.3% level of VaR.

Una aproximación ingenua para acelerar el programa de optimización de carteras usando un algoritmo genético multiobjetivo

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RESUMEN

Los algoritmos genéticos son apropiados cuando los inversores tienen el propósito de obtener la frontera eficiente media-VaR, ya que minimizar el VaR ocasiona que el problema de optimización rentabilidad-riesgo no sea ni convexo ni diferencial. Sin embargo, los algoritmos genéticos son una técnica de optimización que exige mucho tiempo de computación. En este artículo proponemos usar una aproximación *naïve,* consistente en dividir la muestra por cuartiles de riesgo para obtener la frontera eficiente en un tiempo razonable. Nuestros resultados muestran que usando problemas reducidos que sólo consideran un cuartil de los activos podemos explorar la frontera eficiente para un mayor número de niveles de riesgo. Concretamente, la muestra del tercer cuartil permite obtener rápidamente fronteras eficientes con un VaR entre el 1,8 y el 2,5%, mientras que el primer cuartil permite obtener las carteras eficientes con niveles de VaR entre el 1 y el 1,3%.

1. Introduction

Portfolio selection is obtained maximizing expected return and minimizing risk. Nowadays researchers and practitioners are focused on Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) as measures of market risk. VaR of a portfolio is the lowest amount such that the loss will not exceed it with probability $1-\alpha$. CVaR is the conditional expectation of losses above the VaR. VaR and CVaR can be used to balance risk and return. While CVaR can be efficiently minimized using linear programming and non-smoothing techniques (Rockafellar and Uryasev, 2000), minimizing VaR leads to a non-convex and non-differential risk-return problem and smoothing techniques (Gaivoronski and Pflug, 2005) or heuristic optimization techniques need to be applied (Gilli, Këllezi and Hysi, 2006).

In order to avoid the use of smoothing techniques, genetic algorithm (GA) has been used to deal with the problem of minimizing VaR or any other measure that leads to non-convex and

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non-differential risk-return optimization problems. Some examples of application of GAs to the portfolio selection problem are the following: Yang (2006) introduces a GA into a state dependent dynamic portfolio optimization system in order to improve the portfolio efficiency over the classical mean-variance method by reducing the estimation risk. Lin and Liu (2008), Anagnostopoulos and Mamanis (2010) and Baixauli-Soler, Alfaro-Cid and Fernandez-Blanco (2011) use GA for introducing several real constrains to the portfolio selection problems. Ong, Huang and Tzeng (2005) provide an application of multiobjective genetic algorithm to obtain efficient frontiers to improve the accuracy of the mean-variance approach when a small sample is available. Anagnostopoulos and Mamanis (2009) consider integer constrains for the mean-variance model. Subbu, Bonissone, Eklund, Bollapragadaa and Chalermkraivuth (2005) present an optimization approach in which a multiobjective genetic algorithm is combined with linear programming to identify efficient frontiers under multiple risk measures. Also, Baixauli-Soler, Alfaro-Cid and Fernandez-Blanco (2010) report efficient frontiers under multiple risk measures.

One of the benefits of using GAs for multiobjective optimization is that GAs work with a population of individuals (portfolios), which allows us to find several nondominated solution in a single run. Also, GAs are less susceptible than other techniques to the non-convexity of the search space. However, GAs are a time-consuming optimization technique and, when the optimal solution is unknown, the algorithm is stopped when the efficient frontier does not improve significantly.

In this paper, we carry on an analysis of efficient portfolios using VaR and CVaR to quantify the market risk in order to speed up the algorithm. The proposal to speed up the algorithm is a naïve approach based on solving smaller problems. Concretely, firstly, we show differences between efficient portfolios depending on the parameter values used in the multiobjective GA, since these values affect to the computation time. Secondly, we show the good performance of the multiobjective GA comparing its mean-CVaR efficient frontier with the true efficient frontier, which is obtained using the linear programming technique proposed by Rockafellar and Uryasev (2000). We observe that they are overlapped. Thirdly, we analyze the composition and characteristics of the true mean-CVaR efficient frontier in order to propose a naïve approach to obtain mean-VaR efficient portfolios based on a simplification of the portfolio problem splitting the sample by levels of risk. Finally, we compute the efficient frontiers using this naïve approach and we present its advantages and disadvantages.

The remainder of this paper is organized as follows: Section 2 describes the portfolio optimization problem; section 3 explains the implementation of multiobjective GAs; section 4 presents the data and the results, and finally, section 5 reports the main conclusions and the future lines of work.

2. Portfolio Selection Problems

The mean-VaR model uses VaR as a measure of risk. In this paper, we compute VaR using the historical simulation method, which is the most widely used method to do it. This method consists of going back in time and applying current weights to time-series of historical asset returns. By keeping weights at their current values the history of a hypothetical portfolio is reconstructed:

$$R_{p,j} = \sum_{i=1}^{N} w_i R_{i,j} \quad \text{for } j = 1,..,T \qquad [1]$$

where, each historical day is assigned the same probability of occurrence, equal to $1/T$. By sorting in ascending order $R_{p,j}$, VaR is the $R^*_{p,j}$ located in the αT position.

In the mean-VaR model efficient portfolios are the solution of the following problem:

Model 1

$$\begin{aligned} &\underset{w}{Min}\ \ VaR_\alpha(w) = \inf\{r \mid F(r) \geq \alpha\} \\ &\underset{w}{Max}\ \ E(r) = w' r \\ &\text{s. t.} \\ &w'\mathbf{1} = 1 \\ &w_j \geq 0 \ \ \forall j = 1,...,n \end{aligned} \qquad [2]$$

VaR is difficult to optimize for discrete distributions since is non-convex and has multiple local extrema. Mostly, approaches rely on linear approximation of the portfolio risks and assume a joint normal distribution of the underlying parameters (Jorion, 1996, Duffie and Pan, 1997). Optimization requires smoothing or heuristic techniques as those presented in Gaivoronski and Pflug (2005) or Gilli et al (2006). We use a multiobjective genetic algorithm which does not rely on specific assumptions about the distribution of the portfolio return (Subbu et al, 2005, Anagnostopoulos and Mamanis, 2010 and Baixauli-Soler et al, 2011).

On the other hand, CVaR can be defined as the conditional expected loss in case the VaR is exceeded. For general distributions, CVaR is more attractive than VaR since it is sub-additive and convex. On consequence, the problem of minimizing CVaR for finding efficient portfolios is convex.

As for VaR, we compute CVaR using the historical simulation method, whose main advantages are that it is simple to implement and allows for non-normal distribution. Hence, we obtain CVaR from the entire distribution of historical returns and it is estimated by the sample mean of $R_{p,j}$ beyond VaR:

$$CVaR_\alpha = \alpha T^{-1} \sum_{j=1}^{\alpha T} R_{p,j} \qquad [3]$$

To obtain the real mean-CVaR efficient portfolios we consider that CVaR of a random variable $r(w,y)$, which represents the return of the portfolio w under scenarios y, can be calculated by solving a convex optimization problem (Rockafellar and Uryasev, 2000, 2002). The approach characterises $VaR_\alpha(w)$ and $CVaR_\alpha(w)$ in terms of the function $F_\alpha(w, VaR)$:

$$F_\alpha(w,VaR) = -VaR + \frac{1}{\alpha}\int \left[-r(w,y) + VaR\right]^+ p(y)dy \qquad [4]$$

where $[u]^+=u$ for $u\geq 0$ and $[u]^+=0$ for $u\leq 0$. As a function of $VaR_\alpha(w)$, $F_\alpha(w, VaR)$ is convex and continuously differentiable and $CVaR_\alpha(w)$ can be determined minimizing $F_\alpha(w, VaR)$, that is, $CVaR_\alpha(w)=min\, F_\alpha(w, VaR)$. The integral in [4] can be approximated by sampling the probability distribution of y according to its density $p(y)$. If sampling generates T scenarios, y_j j=1,...,T then the approximation is,

$$F_\alpha(w,VaR) = -VaR + \frac{1}{\alpha}\sum_{j=1}^{T} \pi_j \left[-r(w,y_j) + VaR\right]^+ \qquad [5]$$

where π_j is the probability of scenarios y_j and $r(w,y_j) = \sum_{i=1}^{n} w_j r_{ij}$, being r_{ij} the return of asset i under scenario j. Using auxiliaries variables z_j, j=1,…,T the function $F_\alpha(w, VaR)$ can be replaced by the linear function, $F_\alpha(w,VaR) = -VaR + \frac{1}{\alpha}\sum_{j=1}^{T}\pi_j z_j$ and the set of linear constraints, $z_j \geq -r(w,y_j) + VaR,\ \ z_j \geq 0,\ \ j = 1,...,T$. Then investors have to solve the linear problem represented in model 2A. Lim, Sherali and Uryasev (2010) propose an approach to reduce the time consumed for solving the linear problem under a large number of scenarios.

Model 2A

$$MinCVaR_{\alpha}\left(w\right)=\underset{VaR,w,z}{Min}\ -VaR+\frac{1}{\alpha T}\sum_{j=1}^{T}z_{j}$$

s. t.

$$z_{j}\geq-\sum_{j=1}^{T}w_{j}r_{ij}+VaR\quad\forall j=1,...,T$$

$$z_{j}\geq 0\ \ \forall j=1,...,T$$

$$w'r\geq r^{*}$$

$$w'\mathbf{1}=1$$

$$w_{j}\geq 0\ \forall j=1,...,n$$

Model 2B

$$MinCVaR_{\alpha}\left(w\right)=\frac{1}{\alpha T}\sum_{j=1}^{\alpha}R_{p,j}$$

$$\underset{w}{Max}\ \ \mathrm{E}\left(r\right)=w'r$$

s. t.

$$w'\mathbf{1}=1$$

$$w_{j}\geq 0\ \forall j=1,...,n$$

[6]

Model 2A is a linear problem with an exact solution for each r^{*} and Model 2B is the formulation used in the multiobjective algorithm problem. In this case we do not need linearization since a multiobjective genetic algorithm searches the space of solutions. In this case we can compare solutions of Model 2A and 2B in order to evaluate the performance of multiobjective genetic algorithm. When we consider VaR as measure of risk, model 1, we can not evaluate how near is the optimum.

3. Multiobjective Genetic Algorithm Implementation

The GA implementation used in this work is based on ECJ (http://cs.gmu.edu/~eclab/projects/ecj/), a research evolutionary computation system in Java developed at George Mason University's Evolutionary Computation Laboratory (ECLab). For the multi-objective aspect of the optimization the SPEA2 (Strength Pareto Evolutionary Algorithm 2) package of ECJ was used (Zitzler, Laumanns and Thiele, 2001). SPEA2 is an improved version of SPEA which incorporates a fine-grained fitness assignment strategy, a density estimation technique and an enhanced archive truncation method. As most of the multi-objective evolutionary methods it keeps an *archive* where the non-dominated solutions are stored. The size of the archive is set by the user so that if the number of non-dominated solutions is bigger than the archive size the archive is truncated. More details about the multiobjective GA applied to the mean-VaR model can be found in Alfaro-Cid, Baixauli-Soler and Fernandez-Blanco (2011).

The algorithm works as follows:

In step 1 and 2 the archive, *A(g)*, where the non-dominated solutions are stored and the population, *P(g)*, are initialized. *A(0)* is an empty set and *P(0)* is initialized at random.

In step 3 the generation counter g is set to 1 and then the evolution loop starts.

In step 4 and 5 the individuals in the population and the archive are evaluated.

According to this evaluation a new archive is created in step 6 containing all the non-dominated individuals found in the union of the previous archive and the population.

If the size of the resulting archive exceeds the archive size, in step 7 the archive is truncated. This truncation method removes those individuals which are at the minimum distance of another individual. This way the characteristics of the non-dominated front are preserved and outer solutions are not lost.

The termination criterion in step 8 stops the algorithm when the number of generations has been completed.

In step 9 tournament selection with replacement is performed in the archive set in order to fill the mating pool, *M(g)*.

The new population, *P(g)*, is created in step 10 by applying crossover and mutation to the mating pool.

In step 11 the generation counter is increased.

The following pseudo-code details the algorithm main loop:

```
1:   A(0) = ∅
2:   P(0) = init_random();
3:   g = 1;
4:   eval(P(g-1));
5:   eval(A(g-1));
6:   A(g) = save(P(g-1), A(g-1));
7:   truncate(A(g));
8:   if g>g_max then stop;
9:   M(g) = select(A(g));
10:  P(g) = cross&mut(M(g));
11:  g = g+1;
12:  go to Step 4;
```

The control parameters of the GA used are quite standard. The GA is generational. It uses tournament selection with tournament size of 7. The probabilities of crossover and mutation are 1 and 0.05 respectively. The population sizes are 1000 and 2000, the archive sizes are 100 and 200 and the run finishes after 50 and 100 generations. Each individual is encoded as a vector of integers ranging from 0 to 99. Every element of the vector represents the percentage of the budget invested in that particular asset ($w_j^{GA}\leq 0$ j=1,…, n). Therefore, the length of the vector equals the number of assets available in the portfolio. However, the summation of these weights will not be 1, violating the constraint $\sum_{j=1}^{n}w_{j}=1$. This constraint imposes the need of normalizing the vector during the decoding process as follows:

$$\sum_{j=1}^{n}w_{j}=1 \qquad [7]$$

where w_j represents the weight invested in asset j. However, these normalized weights are real values.

4. Data and Empirical Analysis

The data used in this work were extracted from the Bloomberg database. It is a set composed of fifty stocks which belonged to the Eurostoxx 50 index in January 2008. Three stocks with negative expected return in the analysis period were eliminated. We use daily data of these stocks from January 2003 to December 2007. This gives us 1300 observations per stock. We chose daily data instead of monthly data to avoid inaccurate VaR estimates from small samples.

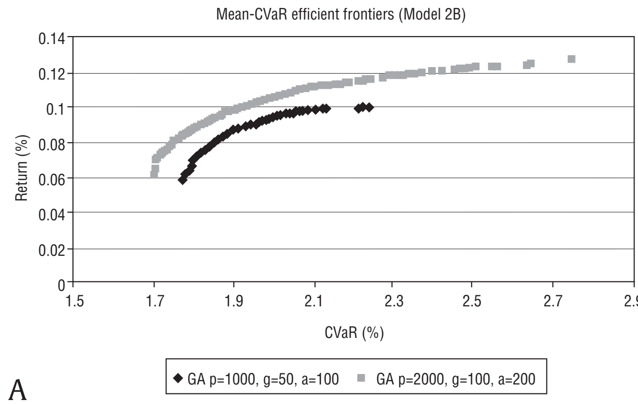
Table 1 reports the descriptive analysis of the data identifying companies by country. It can be observed that the mean daily return is close to zero. This is consistent with computing VaR under the assumption of expected daily return equal to zero. Table 1 reports standard deviation, VaR and CVaR at 95% confidence level for each stock. In order to compute the three risk measures we used historical simulation.

Additionally, we compute Bera Jarque (BJ) statistic to test normality. The BJ statistic has a chi-squared distribution with two degrees of freedom under the null hypothesis that returns are normally distributed. As can be observed the minimum value of the BJ statistic is 59.29 while the critical value is 9.21 for a 1% significance level. Hence normality is rejected in all cases and portfolio risk level ranking is different depending on the measure selected: standard

Table 1
Summary of data statistics.

Country	Company	Mean	SD	$VaR_{95\%}$	$CVaR_{95\%}$	BJ
France	Air Liquide	0.048	1.270	1.928	2.800	175.6
France	Alcatel-Lucent	0.004	2.333	3.472	5.457	640.5
Germany	Allianz SE	0.039	1.853	2.911	4.398	985.6
Italy	Assicurazioni Generali SpA	0.041	1.243	1.771	2.832	308.1
France	AXA SA	0.054	1.861	2.898	4.238	344.2
Spain	BBVA SA	0.043	1.349	2.141	3.126	213.9
Spain	Banco Santander SA	0.059	1.360	2.188	3.202	300.6
Germany	BASF SE	0.074	1.429	2.061	3.087	623.4
Germany	Bayer AG	0.087	1.993	2.551	4.161	3557.7
France	BNP Paribas	0.047	1.487	2.371	3.260	425.4
France	Carrefour SA	0.015	1.432	2.171	3.254	1700.9
France	Cie de Saint-Gobain	0.060	1.608	2.580	3.629	2694.3
France	Credit Agricole SA	0.036	1.485	2.210	3.322	112.9
Germany	Daimler AG	0.056	1.650	2.717	3.630	24340.5
Germany	Deutsche Bank AG	0.048	1.559	2.494	3.415	1227.9
Germany	Deutsche Boerse AG	0.151	1.669	2.434	3.577	2190.6
Germany	Deutsche Telekom AG	0.011	1.465	2.063	3.529	20488.1
Germany	E.ON AG	0.099	1.418	2.074	3.163	6325.7
Italy	Enel SpA	0.042	0.973	1.449	2.237	355.6
Italy	ENI SpA	0.036	1.147	1.855	2.679	918.8
Belgium	Fortis	0.016	1.770	2.634	4.339	678.4
France	France Telecom SA	0.036	1.641	2.339	3.621	238.27
France	Groupe Danone	0.049	1.222	1.705	2.470	1191.2
Spain	Iberdrola SA	0.087	1.094	1.523	2.277	3887.1
Netherlands	ING Groep NV	0.035	1.851	2.827	4.389	8069.3
Italy	Intesa Sanpaolo SpA	0.070	1.535	2.191	3.323	147.1
Netherlands	Koninklijke Philips Electronics NV	0.037	1.828	2.800	4.075	548.9
France	L'Oreal SA	0.021	1.347	2.021	3.032	871.2
France	LVMH SA	0.054	1.369	2.166	2.984	1032.8
Germany	Muenchener Rueckversicherungs AG	0.011	1.845	2.738	4.533	2398.5
Finland	Nokia OYJ	0.038	2.019	2.887	4.755	170.7
France	Renault SA	0.056	1.721	2.745	3.842	459.7
Spain	Repsol YPF SA	0.048	1.230	1.923	2.857	435.1
Germany	RWE AG	0.100	1.535	2.273	3.456	944.7
France	Sanofi-Aventis SA	0.005	1.471	2.235	3.426	7508.2
Germany	SAP AG	0.043	1.719	2.430	3.798	539.9
Germany	Schneider Electric SA	0.054	1.442	2.258	3.241	59.29
Germany	Siemens AG	0.069	1.617	2.535	3.569	629.3
France	Societe Generale	0.042	1.550	2.416	3.537	6005.0
France	Suez SA	0.076	1.952	2.657	4.582	668.3
Spain	Telefonica SA	0.077	1.198	1.853	2.664	2330.4
France	Total SA	0.038	1.240	2.143	2.774	339.6
Italy	UniCredit SpA	0.029	1.273	2.007	3.033	3106.8
Netherlands	Unilever NV	0.019	1.227	1.776	2.942	399.8
France	Vinci SA	0.103	1.404	1.947	2.919	555.3
France	Vivendi	0.050	1.639	2.439	3.792	2778.3
Germany	Volkswagen AG	0.107	1.773	2.717	3.789	200813.2

Mean: the average daily return; SD: standard deviation; $VaR_{95\%}$: the 95% 1-day VaR, and $CVaR_{95\%}$: the 95% 1-day CVaR are expressed in percentage and are computed with daily returns from January 2003 to December 2007. BJ is the Bera-Jarque statistic.



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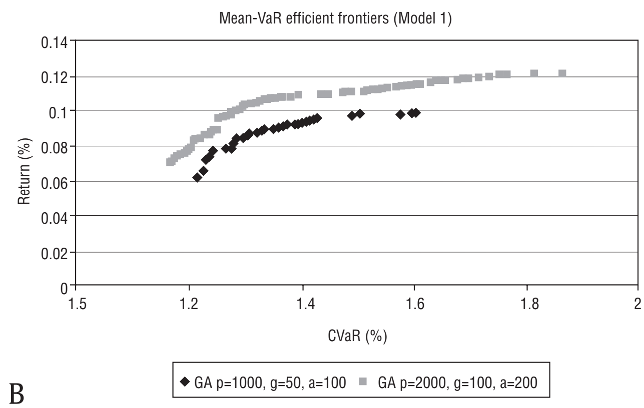

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Figure 1. Mean-CVaR and Mean-VaR efficient frontier computed with different parameter values.

deviation, VaR or CVaR. Also, Table 1 reports standard deviation, VaR and CVaR at 95% confidence level. For example, we can find Siemens AG with 1.617% of deviation, 2.535% of VaR and 3.569% of CVaR while SAP AG has higher deviation 1.719%, lower VaR 2.43% and higher CVaR 3.798%. These are only examples since they are not representative of the subset of efficient portfolios.

In Figure 1 we show graphically the differences among efficient portfolios depending on the parameter values used in the multiobjective GA since these values affect significantly to the computation time.

In Model 1, the parameter values of the multiobjective GA have been selected by evaluating how changes in parameter values affect the efficient frontier. Figure 1 shows the results when Model 1 (the mean-VaR problem) and Model 2B (mean-CVaR problem) are solved using multiobjective GAs for different parameter values. In both models, firstly the population size, the archive size are fixed equal to 1000 and 100, respectively, and the run finishes after 50 generations. Secondly, the parameter values are chosen equal to 2000 for the population size and 200 for the archive size, and the run finishes after 100 generations. As Figure 1 shows changes in the parameter values implied a significant improvement in the efficient frontier. It must be highlighted that the biggest improvement is due to the increase in the number of generations and not to changes in the archive size or the population: the results are not presented to preserve simplicity, but similar results have been obtained when we fixed a population size of 1000, an archive size of 100 and a number of generations of 100. However, a significant improvement in mean-CVaR efficient frontier is not obtained if the number of generations increases from 100 to 200.

On the other hand, we are also interested in analyzing the performance of the multiobjective GA versus linear programming by comparing the mean-CVaR efficient frontier from Model 2B with that obtained the using linear programming technique proposed by Rockafellar and Uryasev (2000), represented by Model 2A.

Figure 2 shows the CVaR-efficient frontiers computed using LP and multiobjective GAs (Model 2A and 2B). As it can be observed they are overlapped, what reflects an excellent performance of the multiobjective GA. This indicates that multiobjective GAs can solve mean-CVaR problems without using a linear transformation. The parameter values used in the multiobjective GAs were 2000 for the population size and 200 for the archive size, and the run finished after 100 generations. As we pointed out before an increase in the values of these parameters improves the solution insignificantly while increases the computation time significantly.

In order to infer a strategy to speed up the multiobjective GA, we analyze the portfolio composition and characteristics of the efficient portfolios. Figure 3 and Table 2 contain a description of the efficient portfolio composition and characteristics from the mean-CVaR optimal frontier.

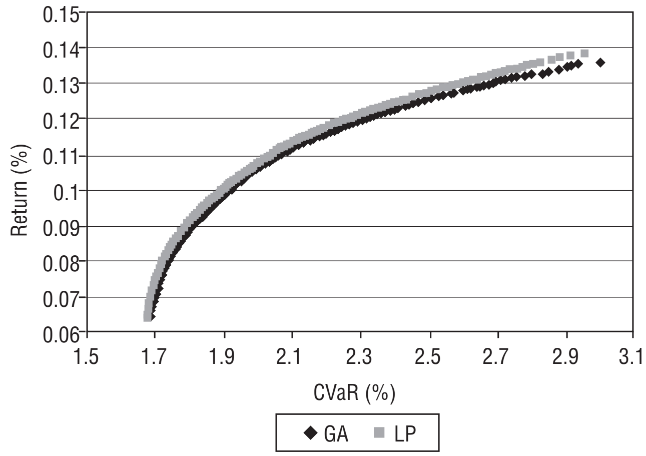

Figure 2. Mean-CVaR efficient frontier computed with GA and LP. Note: Optimization results for the mean-CVaR optimization problem computed with LP (linear programming) and multiobjective GA (multiobjective genetic algorithm).

The 483 efficient portfolios analyzed are plotted in Figure 2. As Figure 3 shows there are some assets which are invested in the majority of the efficient portfolios. In particular, asset 16 (Deutsche Roerse), asset 18 (E.ON AG), asset 45 (Vinci) and asset 47 (Volkswagen) are in the 483 efficient portfolios, asset 24 (Iberdrola) is in the 98% of the efficient portfolios and asset 34 (RWE AG) in approximately the 90% of them. The mean presence of the rest of assets in the efficient portfolios is around the 10%. Table 2 shows that theses assets are the assets that have the highest return by unit of risk and hence they dominate other assets.

Table 2
Characteristics of risk and return of most invested assets.

Asset Number	Company	Return	$CVaR_{95\%}$	Ranking R	Ranking CVaR	Ranking R/CVaR
16	Deutsche Boerse AG	0.151	3.577	1	30	1
18	E.ON AG	0.099	3.163	5	17	4
24	Iberdrola SA	0.087	2.277	7	2	2
34	RWE AG	0.100	3.456	4	26	5
45	Vinci SA	0.103	2.919	3	10	3
47	Volkswagen AG	0.107	3.789	2	34	7

Note: Ranking R is the position of the asset when all individual assets are arranged in descending order of return; Ranking CVaR is the position of the asset when they are arranged in ascending order of CVaR; Ranking R/CVaR is the position of the asset when they are arranged in descending order of R/CVaR.

Table 3 presents the results when the 483 efficient portfolios are analyzed depending on the risk quartile they belong. The mean value of assets in the efficient portfolios and the associated standard deviation decrease from the first quartile to the fourth: in particular, from 18 (mean value) and 10.6 (standard deviation) in the first quartile to 6 and 1.6 in the fourth. Also the maximum number of assets decreases from 43 to 19 and the minimum from 8 to 4.

For the set of assets that belong to each risk quartile, Table 4 reports the percentage of times that they are invested in the efficient portfolios belonging to each quartile of risk. Table 4 shows that assets are not distributed uniformly through the efficient portfolios of each quartile of risk. For instance, it can be observed that the assets that belong to the first quartile of risk are invested in the 37,5%, 16,9%, 31,61% and 13,9% of the efficient portfolios of the first, second, third and fourth quartile of risk, respectively.

According with this evidence, it could be useful to use the risk measure to split the sample and to solve the multiobjective GA for each sub-sample in order to speed up and improve the algorithm. Probably the first idea to reduce the number of assets could be to use low-risk assets to search for low-risk portfolios and to use high-risk assets to search for high-risk portfolios.

Figure 4 reports the results for the mean-VaR efficient frontier for Model 1, using the whole sample of assets and the sub-samples constructed with the assets that belong to each quartile of risk. In Figure 4 it can be seen that the efficient frontier obtained with the whole

Table 3
Characteristics of efficient portfolios classified by quartile of risk.

	Q1	Q2	Q3	Q4
Mean Return	0.080	0.097	0.111	0.126
Mean CVaR	1.728	1.873	2.067	2.485
Number of assets				
Mean	18.082	14.545	8.525	6.157
SD	10.676	8.279	5.643	1.668
Max	43	41	39	19
Min	8	8	5	4

Table 4
Distribution of the investment by asset level of risk and efficient portfolio level of risk.

	Q1	Q2	Q3	Q4
Asset Q1-CVaR	37.563	16.925	31.610	13.900
Asset Q2-CVaR	35.215	13.577	34.029	17.178
Asset Q3-CVaR	34.854	16.660	42.327	6.157
Asset Q4-CVaR	33.197	17.332	46.258	3.211

Note: For the set of assets that belong to each risk quartile, the percentage of times that they are invested in the efficient portfolios belonging to each quartile are reported.

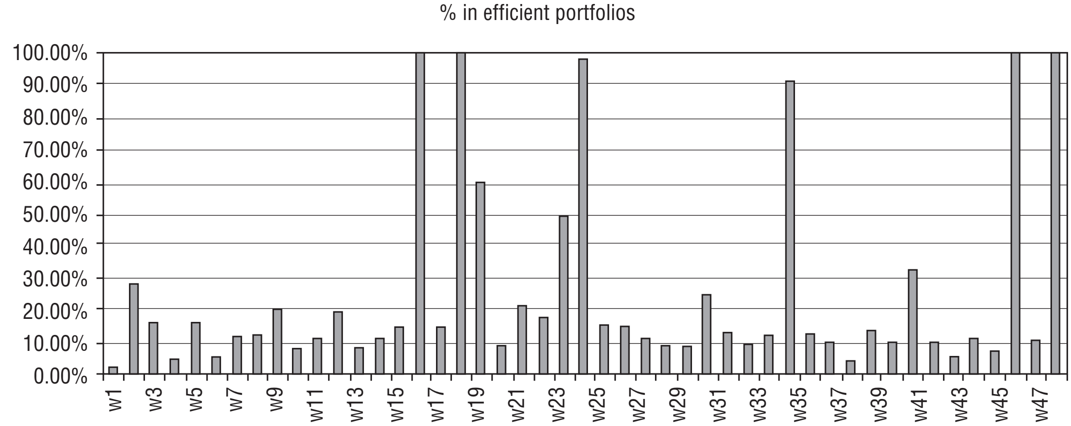

Figure 3. Composition of Mean-CVaR optimal portfolios.
Note: X-axis represents each of the assets considered as available in the market to obtain the mean-CVaR efficient portfolios. Y-axis represents the percentage of efficient portfolios in which the asset has a positive weight.

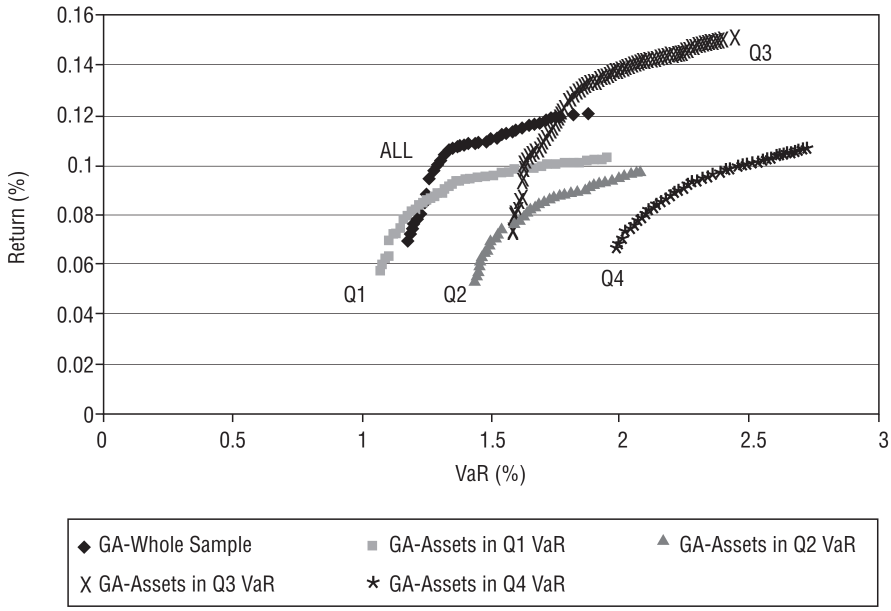

Figure 4. Mean-VaR efficient frontiers for different sets of assets.

sample is restricted to values of VaR over 1% and below 2%. The efficient portfolios out of this range are not obtained because the multiobjective GA searches in the space of solutions. In order to solve this problem the population size, the number of generations and the archive size could be increase, what usually implies an increase in the computation time. By contrast, the reduced problems which consider a quartile of the assets allow us to explore the efficient frontier for a bigger range of risk values. Concretely, the third quartile allows us to obtain quickly efficient frontiers from 1,8% to 2,5% level of VaR, and the first quartile of assets allows us to obtain quickly frontiers from 1% to 1,3% level of VaR. In sum up, to use a sample split by risk is a naïve approach that allows us to obtain complete efficient frontiers in a reasonable computation time.

5. Conclusions

In this paper, it has been shown that multiobjective GAs are appropriated when investors have the purpose of obtaining the mean-VaR efficient frontier. GAs are a time-consuming optimization technique, and when the optimal solution is unknown, the algorithm is stopped when the efficient frontier does not improve significantly. In our work we have evaluated the solution improvement which is obtained when a naïve approach is used in a multiobjective GA. This approach consists on splitting the sample by quartiles of risk due to the fact that the number of assets which are in an efficient portfolio is not uniformly distributed across the four quartiles. Our results show that there is a new space of solutions explored when the naïve approach is used without increasing the time of computation.

To develop new approaches to reduce time of computation in multiobjective GAs as the presented in this paper is extremely useful to obtain optimal portfolios under a measure of risk which leads to a non-convex and non-differential risk-return problem, as it happens with mean-VaR problem. Also they are useful when real constraints and non-linear cost structure are included in the selection portfolio problem even though a convex measure of risk is used or when derivatives which are no linear instruments are considered, since time of computation of heuristic methods increases significantly in all these cases.

Among the possible future directions of research, it is interesting identify more characteristics of assets in extreme efficient portfolios, such as high risk and return efficient portfolios, and low risk and return efficient portfolios, in order to reduce time of computation driven the algorithm by the space of solutions.

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